ECO 328: The Neutral Rate

These notes follow:

Holston, K., Laubach, T and Jon Williams. "Measuring the Natural Rate of Interest: International Trends and Determinants."*Journal of International Economics* 108, May 2017.

We begin with some background. The real natural rate of interest is the real interest rate that would exist absent short-term frictions, such as those caused by sticky prices. Under reasonable assumptions, we may think of the natural rate as identical to the steady state rate.

Now consider a simple growth model. Profit maximization implies that the steady state real interest rate is marginal product of capital less the depreciation rate:

$$r* = \alpha A(\frac{K}{L})^{\alpha-1} - \delta \tag{1}$$

assuming a Cobb-Douglas production function. This tell us a few thing about the natural rate of interest.

1. In the short-run, an increase in TFP (AP, increases the neutral rate by raising productivity and making firm more willing to borrow to finance capital investment. If, for the sake of argument, AI leads to a productivity boom, we would expect it to raise the neutral rate.

2. Longer term, we must be careful about describing TFP's impact on r*. Capital is endogenous and we expect that higher TFP will lead to a higher capital stock.

3. All else equal, a higher ratio of capital to worker leads to reduced marginal productivity and a lower r^*. And that makes me a sad panda.

4. A higher depreciation rate lowers the real rate. Historically, the depreciation rate of capital has been steady over time. We can speculate that AI could change the rate of depreciation.

The relationhip between r* and a Central Bank' neutral rate is straightforward. We simply add the inflation target, 2% for the Fed, to get the neutral rate.

Maximum Liklihood etimation

The econometric analysis that follows often relies on maximum likelihood estimation (MLE). This is an alternate approach to OLS and related techniques. Rather than minimizing the sum of squared errors, we instead maximum a function. For ome linear specifications, the two approaches are identical. MLE, however, easily extends to non-linear models.

Consider a simple example. We assume that a variable is drawn from a normal distribution with mean μ and variance σ^2. The normal distributiin is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp^{\frac{x-\mu}{\sigma}} \tag{2}$$

Suppose that we observe three draws where the variable equals 1, 2, and 3. The likelihood of these three draws is then:

$$\frac{1}{\sqrt{2\pi\sigma^2}}[exp^{\frac{1-\mu}{\sigma}} exp^{\frac{2-\mu}{\sigma}} exp^{\frac{3-\mu}{\sigma}}] \tag{3}$$

MLE consists of using an algorithm to pick μ and σ to maximize this likliehood function.

The trick to MLE is writing out the appropriate liliehood function. For a standard mulivaroaye regression, we can use the following:

$$\ln L(\beta, \sigma^2, y, X) = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log(\sigma^2) - \frac{1}{2\sigma^2}(y - X\beta)^T(y - X\beta) \tag{4}$$

where β is the vector of regression coefficients and σ^2 is the variance-covariance matrix. Note that maximizing the log of the likelihood function is the same as maximizing the likelihood function, but is often easier.

Model

This is an example of stryctural work where the authors are estimating an explicit theoretical model. The model may be thought of as a more general version of Gali.

$$\tilde{y}_t = a_{y_1}\tilde{y}_{t-1} + a_{y_2}\tilde{y}_{t-2} + \frac{a_r}{2} + \sigma^2_{j=1}(r_{t-j} - r^*_{t-j}) + e_{y,t} \tag{5}$$

$$\pi_t = b_\pi \pi_{t-1} = (1 - b_\pi)\pi_{t-2,4} + b_y \tilde{y}_{t-1} + e_{\pi,t} \tag{6}$$

where $\pi_{t-2,4}$. These two equations serve the same purpose as the New Keynesian Phillips Curve and the Euler Equation. We can think of the expectations of Gali as depending on lags to yield this model. Furthermore:

$$r^*_t = g_t + z_t \tag{7}$$

where g_t is trend-GDP growth.

$$y^*_t = y^*_{t-1} + g_{t-1} + e_{y,t} \tag{8}$$

$$g_t = g_{t-1} + e_{g,t} \tag{9}$$

$$z_t = z_{t-1} + z_{g,t} \tag{10}$$

The econometric challenge is how to go from observables (output and inflation) to unobservables, such as the natural rates of output and interest.

Estimation

The authors use a technique called the Kalman filter to obtain a time-series for the natural rate of output. This is similar to de-trending the GDP series. This yields the variances of y^* and g. They then use MLE to estimate the remaining equations.

Results

Rather than focus on the results preented in the 2017 paper, we will focus on the latest results, which are updated each quarter:

1. As of 4Q2025, the real natural rate of interest for the U.S. was 0.9$, implying a neutral rate of 2.9%. Note that Jon Williams, one of the authors, is president of the New York Fed, a permanent voting member of the FOMC. This means that current (2Q2026) U.S. monetary policy remains modestly restrictive.

2. The neutral rate has fallen dramatically in recent decades. It was above 7$ in the 1960s and has declined steadily since. If we think of neutral as roughly average, this explains the clear downward trend in the Federal Funds rate. It also may help explain why bond market's seem to expect a higher neutral rate. They may be taking an average of past Fed Funds rates.

3. The current estimate of trend-GDP growth is 2.5%. Trend-GDP growth slowed from the 1960s through about 2012 but has slightly recovered since. The paper itself is agnostic about what has affected trend-growth. Technology and population growth dynamics have certainly played a role. Higher trend GDP growth is positively correlated either neutral rate, but there are other factors as well. This makes it risky to assume that factors such as AI are likely to lead to a higher neutral rate.

4. For the Eurozone, the neutral rate is just 2%. This is partly because trend-DP growth is also very low at just 1.3%. For Canad, the neutral rate is much higher, at 3.7%, but trend-GDP growth is 1.9%.