Money in a Classical Model

These notes follow Gali Ch. 2. The model that we develop is a stepping stone toward our New Keynesian model. It lacks two things that make the latter Keynesian; sticky prices and monopolistic competition.

This model illustrates two important concepts. First, it includes a money market where money provides utility to households. Second, it models a bond market.

The text includes cases with and without additively separate utility. We will focus on the former:

$$Max_{B_t,C_t,N_t} \;\; E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, \frac{M_t}{P_t}, N_t) \tag{1}$$

$$lim_{T\to\infty} \;\; E_t[M_t + B_t] > 0 \tag{2}$$

There are several aspects of this problem that merit discussion:

1. B_t are riskless one-period bonds. Households purchase them in period t at the price Q_t and they pay 1 in period $t+1$. This bond market is important. Recall that our growth models obtained an Euler Equation through a capital market. For simplicity, this model neglects capital. The Euler Equation instead comes from the bond market.

2. The representative household is one of an infinite and identical # of such households. It thus takes all process as given.

3. Because all households are the same, and there is no government, $B_t = 0$, in equilibrium. We cannot, however, impose this condition until after optimization. The potential to buy bonds out of equilibrium (even though one would never want to) is important.

4. Households obtain utility from their real money holdings. The motivation behind this assumption is that money is a social convention that provides convenience. I choose to hold money because I know that everyone else accepts it as part of transactions and it is easier than barter.

The monetary economics literature diverges here. If we accept this logic, then we can proceed with money in the utility function (or a related modeling device). Some monetary economists, however, find this logic unappealing. They thus prefer to think more deeply about why households hold money. We will take the former approach, which yields ample policy implications. The latter is more conceptual and less informative about policy. The household's budget constraint is:

$$M_t + P_t C_t + Q_t B_t \leq B_{t-1} + w_t N_t - T_t + M_{t-1} \tag{3}$$

Most of these variables are self explanatory. T_t, however, represents lump sum transfers that the household takes as given (they thus disappear during optimization). P_t is the price of the consumption good.

We further assume the following instantaneous utility function:

$$u(*) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{(\frac{M_t}{P_t})^{1-\nu}}{1-\nu} - \frac{N_t^{1+\psi}}{1+\psi} \tag{4}$$

Optimization

We optimize via argument:

1. Suppose that the household increases consumption expenditures by one very small unit. It is then able to buy $\frac{1}{P_t}$ units of the consumption good. This converts to $\frac{u_{c,t}}{P_t}$ units of utility in period t where $u_{c,t}$ is the marginal utility of consumption.

2. All else, equal, the household must reduce its bond holdsings by $\frac{1}{Q_t}$ units.

3. In period $t+1$ the household therefore expects to reduce its consumption by $\frac{1}{Q_t P_{t+1}}$ units.

4. The discounted utility loss from #3 is $\frac{\beta u_{c,t+1}}{Q_t P_{t+1}}$.

5. If such an experiment, or its opposite yields a utility gain, then the household cannot be optimizing. It must thus be the case that optimality sets:

$$Q_t = E_t \left[\frac{u_{c,t+1} P_t}{u_{c,t} P_t} \right] = E_t \left[(\frac{C_{t+1}}{C_t})^{-\sigma} \frac{1}{\Pi_{t+1}} \right] \tag{5}$$

where $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$.

Labor supply can be derived through a similar exercise:

1. Suppose that the household increases its labor supply by one very small unit. It thus obtains W_t which results in $\frac{W_t}{P_t}$ additional units of consumption.

2. The household thus obtains $\frac{W_t u_{c,t}}{P_t}$ units of utility.

3. The household also obtains the marginal disutility of labor, $u_{n,t}$ which is negative.

4. It must be the case that these effects sum to zero:

$$\frac{-u_{n,t}}{u_{c,t}} = \frac{W_t}{P_t} \tag{6}$$

or, for our specific utility function from (4):

$$C_t^{\sigma} N_t^{\psi} = \frac{W_t}{P_t} \tag{7}$$

Finally, we do a similar exercise to obtain money demand:

1. Suppose that the household increases its real money holdings by one very small unit in period t. It thus increases its utility by $u_{m,t}$.

2. All eslse equal, the household reduces its consumption in period t by one unit, resulting in a loss of utility equal to $u_{c,t}$.

3. All else equal, the household may increase its consumption by $E_t[\frac{1}{\Pi_{t+1}}]$ in period $t+1$. This results in a discounted expected utility gain equal to $\beta E_t[\frac{u_{c,t+1}}{\Pi_{t+1}}]$.

4. Setting the sum of these terms equal to zero yields:

$$u_{m,t} - u_{c,t} + \beta E_t[\frac{u_{c,t+1}}{\Pi_{t+1}}] = 0 \tag{8}$$

Using (5), we can re-write (8):

$$u_{m,t} - (1 - Q_t) u_{c,t} = 0 \tag{9}$$

Or

$$\frac{u_{m,t}}{u_{c,t}} = 1 - e^{-i_t} \tag{10}$$

where $i_t = lnQ_t^{-1}$, the nominal interest rate. For our utility function:

$$\frac{M_t}{P_t} = C_t^{\frac{\sigma}{\nu}}(1-e^{-i_t})^{\frac{-1}{\nu}} \tag{11}$$

Firm Optimization

The production side of the economy is easier. We assume: 1) perfect competition, and 2) fixed capital so that $K = 1\ \forall t$. The latter is just for simplicity.

We assume the following production function:

$$Y_t = A_t N_t^{1-\alpha} \tag{12}$$

where A_t, as always, is TFP. The representative firms problem is right out of ECO 101:

$$Max_{N_t}\ \ P_t Y_t - W_t N_t \tag{13}$$

Optimization sets the marginal product of labor equal to the real wage:

$$\frac{W_t}{P_t} = (1-\alpha)A_t N_t^{-\alpha} \tag{14}$$

For the remainder of these notes, we will use lower case notation to indicate logs. Taking logs of (14) yields:

$$w_t - p_t = ln(1-\alpha) + a_t - \alpha n_t \tag{15}$$

Equilibrium

We have the following endogenous variables: $W_t, P_t, Y_t, C_t, N_t, B_t, M_t, \Pi_t$, and i_t. Our equations are (3), (5), (7), and (11)-(13). We are thus three equations short. One of them is the observation that, in equilibrium, bond holdings equal zero: $B_t = 0$. Another is the observation that without capital, all output is consumed: $Y_t = C_t$.

The final equation, will be the monetary policy rule. For now, we ignore this. It turns out that we can solve for output and employment independent of monetary policy.

First, take logs of (12):

$$y_t = a_t + (1-\alpha)n_t \tag{16}$$

Now take logs of (7):

$$w_t - p_t = \sigma y_t + \psi n_t \tag{17}$$

We can then combine (15) and (17):

$$ln(1-\alpha) + a_t - \alpha n_t = \sigma y_t + \psi n_t \tag{18}$$

Now use (16), the production function, to eliminate y_t from (18):

$$ln(1-\alpha) + a_t - \alpha n_t = \sigma a_t + \sigma(1-\alpha)n_t + \psi n_t \tag{19}$$

or

$$n_t = \frac{1-\sigma}{\sigma(1-\alpha)+\psi+\alpha}a_t + \frac{ln(1-\alpha)}{\sigma(1-\alpha)+\psi+\alpha} \tag{20}$$

we can then insert (20) into (16):

$$y_t = \frac{1+\psi}{\sigma(1-\alpha)+\psi+\alpha}a_t + \frac{(1-\alpha)ln(1-\alpha)}{\sigma(1-\alpha)+\psi+\alpha} \tag{21}$$

We have thus solved for output, consumption (because output equals consumption), and labor. We now seek to solve for the real interest rate, the cost of borrowing controlling for inflation. The relationship between the real and nominal interest rate is given by the Fisher Equation:

$$r_t \equiv i_t - E_t[\pi_{t+1}] \tag{22}$$

Now take logs of (5), the Euler Equation:

$$y_t = E_t[y_{t+1}] - \sigma^{-1}(i_t - E_t[\pi_{t+1}] - \rho) \tag{23}$$

where $\rho = -ln(\beta)$. Note that (22) carries the expectations operator through. This implies that we are assuming that $E[XY] = E[X] * E[Y]$. Equation (22) is thus not literally an Euler Equation but is instead a *log-linear approximation*. We will see several more linear approximation of non-linear models going forward.

We can re-write (23) using the Fisher Equation, (22):

$$r_t = \sigma E_t[\Delta y_{t+1}] + \rho \tag{24}$$

We now consider how agents form rational expectations. This entails assuming that agents know that(21) describes how output evolves. If they know this then they will expect the following:

$$E_t[\Delta y_{t+1}] = \frac{1+\psi}{\sigma(1-\alpha)+\psi+\alpha} E_t[\Delta a_{t+1}] \tag{25}$$

and inserting (25) into (24) yields:

$$r_t = \frac{\sigma(1+\psi)}{\sigma(1-\alpha)+\psi+\alpha} E_t[\Delta a_{t+1}] \tag{26}$$

Finally, it is direct from (17) that the real wage also depends only on TFP.

We have now shown that output, consumption, the real wage, employment, and the real interest rate are independent of monetary policy. Monetary policy is thus said to be *neutral*. This is a common feature of models in the classical tradition. It shows that we need to add Keynesian features, especially nominal rigidities, for monetary policy to be able to stabilize real variables.

Monetary Policy

We assume that the monetary authority has the ability to set the nominal interest rate i_t. This view is more in line with how actual monetary policy works than assuming that it chooses M_t. To close the model, we assume that the monetary authority uses a monetary policy rule that links i_t to the models other variables. Of course, we do not believe that the Fed, for example, literally uses such a rule. But we do think that a rule, appropriately chosen, can approximate the Fed's choices.

Suppose that the monetary authority uses the following rule:

$$i_t = \rho \tag{27}$$

This rule simply sets the nominal interest rate as a constant. Presumably, the FOMC then goes home and drinks Scotch.

The analysis that follows is challenging. Our goal is always the same, to represent the price level (which is the only thing that monetary policy affects here) into a system of difference equations (in this case a univariate difference equation).

First note that $\pi_t = ln(\frac{P_t}{P_{t-1}}) = p_t - p_{t-1}$. We can then re-write the Fisher Equation as:

$$E_t[p_{t+1}] = p_t + \rho - r_t \tag{28}$$

This is something that we know how to deal with. It is a single difference equation with a constant and an exogenous error term. r_t is exogenous because policy changes that affect the price cannot effect the real interest rate.

For $\rho > 0$, the price level will grow without bound. But this is economically reasonable because the price level is a nominal variable. To finish off the solution, we define $\zeta_{t+1} = p_{t+1} - E_t[p_{t+1}]$ as a sunspot. It is an expectational error that may be tied to any extraneous variable. We then re-write the solution as:

$$p_{t+1} = \zeta_{t+1} + p_t + \rho - r_t \tag{29}$$

This is considered an example of a failed policy. Because any random variable may be interpreted as ζ, this solution allows the public's extraneous expectations to be self-fulfilling. Suppose, for example, that the sunspot depends on China's budget deficit. We are then allowing a variable that should not affect the model, to have real effects anyway. This is known as *indeterminacy*. Avoiding it is a major goal of current monetary policy.

Now consider a more general, and more realistic, monetary policy rule:

$$i_t = \rho + \phi_\pi \pi_t \tag{30}$$

where $\phi_\pi \geq 0$. This rule implies that the monetary authority is "leaning against the wind." When inflation is high, it seeks to reduce it by raising interest rates. Inserting (30) into the Fisher Equation yields:

$$E_t[\pi_{t+1}] = \rho + \phi_\pi \pi_t - r_t \tag{31}$$

or

$$\pi_t = \frac{E_t[\pi_{t+1}] + r_t - \rho}{\phi_\pi} = \frac{E_t[\pi_{t+1}}{\phi_\pi} + \frac{\hat{r}_t}{\phi_\pi} \tag{32}$$

Because inflation is a real variable, we will only consider cases where π_t remains bounded. To solve for inflation, we iterate forward: re-dating (32) yields:

$$E_t[\pi_{t+1} = \frac{E_t[\pi_{t+2}]}{\phi_\pi} + \frac{\hat{r}_{t+1}}{\phi_\pi} \tag{33}$$

Notice that in re-dating (32) to obtain (33), we have not re-dated the time subscripts on the expectational operators. This is due to the *law of iterated expectations*. This states that my expectation of tomorrows expectation of X is just my current expectation of X. In other words, I never expect to change my mind. Inserting (33) into (32):

$$\pi_t = \frac{\hat{r}_t}{\phi_\pi} + \frac{E_t[\hat{r}_{t+1}]}{\phi_\pi} + \frac{E_t[\pi_{t+2}]}{\phi_\pi} \tag{34}$$

and if we keep iterating j of times, a pattern emerges:

$$\pi_t = \phi_\pi^{-(j+1)} E_t[\pi_{t+j}] + \sum_{k=0}^{j} \phi_\pi^{-(k+1)} E_t[\hat{r}_{t+k}] \tag{35}$$

We now consider two cases. First, suppose that the monetary authority chooses $\phi_\pi > 1$. In this case, as we evaluate (5) as $j \to \infty$:

$$\pi_t \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t[\hat{r}_{t+k}] \tag{36}$$

The key result here is that equilibrium is unique, a condition known as *determinacy*. Inflation does not depend on sunspots. This is important because sunspots add volatility to the price level which undermines the monetary authority's goal of stabilization. This is known a s the *Taylor Condition*. Although its exact form varies, it generally states that the monetary authority must be sufficiently aggressive in cracking down on inflation to ensure a unique equilibrium.

Now suppose that the monetary authority fails to satisfy the Taylor Condition. The solution is then simply:

$$\pi_{t+1} = \phi_p i \pi_t - \hat{r}_t + \zeta_{t+1} \tag{37}$$

Once again the solution depends on potentially de-stabilizing sunspots. Equation (37) offers one (there are others) explanation for the volatility of the 1970s. There is some empirical evidence that the Fed failed to satisfy the Taylor Condition. If so, then this may have allowed for sunspots to destabilize prices (as seen in this model), as well as output (a result we will have to wait until Chapter 3 to see).

Optimal Monetary Policy

Having solved for the Taylor Condition, we conclude by solving for the best monetary policy rule. Recall the households money demand equation:

$$\frac{u_{m,t}}{u_{c,t}} = 1 - e^{-i_t} \tag{38}$$

Now consider how a social planner, who does not care about prices, would choose their money demand. We are assuming that money is costless to produce. So to set the marginal cost equal to the marginal benefit, we set:

$$U_{m,t} = 0 \tag{39}$$

Optimal monetary policy equates (38) with (39). This requires that $i_t = \rho$.

Implementing such an outcome is non-trivial. Suppose, for example, that the monetary authority tries to use (27). This yields $\pi_t = -\rho$, which suggests that a slight deflation is optimal (assuming that rho is just above zero). This is known as *Friedman's Rule*.

The problem is that while the optimal allocation is an equilibrium, it is just one of many. We saw this when we showed that this rule gives us indeterminacy.

Finding a rule that gives the optimal allocation as a unique equilibrium requires cleverness.

$$i_t = \phi(r_{t-1} + \pi_t) \tag{40}$$

where $\phi > 1$.

We next re-date (40) and take expectations:

$$E_t[i_{t+1}] = \phi(r_t + E_t[\pi_{t+1}]) \tag{41}$$

Inserting (41) into (40):

$$i_t = \frac{E_t[i_{t+1}]}{\phi} \tag{42}$$

Iterating forward:

$$i_t = \frac{E_t[i_{t+j}]}{\phi^j} \tag{43}$$

Evaluating as $j \to \infty$, we get $i_t = 0$ as a unique equilibrium. Optimality in this setting requires a slight deflation. We should not take this too seriously. The model does not have credit market imperfections that suggest avoiding deflation is a very important policy objective. It is the first of many results, however, that suggest that near price stability is desirable.

The public worries more about unemployment than inflation. To some, there is a disconnect between this desire and the Fed's apparent focus on price stability. In general, as one studies more monetary economics, we find more and more results which suggest that price stability is the best way to achieve output/unemployment stability.