

## Economic Growth<sup>1</sup>

These notes are divided into three parts. First, they discuss some important observations about growth rates over time and across countries. Second, they summarize a few of the most important drivers of growth that economists largely agree on. Third, they discuss our first formal macroeconomic model of the semester, the classic Solow Model of Growth. We will analyze this model entirely through graphs.

### Some Observations About Growth

1. Sustained growth is a relatively recent phenomenon. The human race is about 200,000 years old. For most of its existence, humanity has existed close to *subsistence*. This is the level of output needed to just survive (several hundred dollars per year, per capita). At this level, temporary reductions in output may push society into famine.

Despite considerable technological advancement, most of the earth's population remained close to subsistence until just a few centuries ago. This shows that the annual per-capita growth rate was very close to 0. This led to the *Malthusian* view of economics. This claims that any surplus output will be absorbed by additional population so that per capita output remains close to subsistence. This bleak outlook earned economics the nickname "the dismal science." The Malthusian outlook has not held up well in recent centuries. There are, however, some neo-Malthusians who expect growth to be temporary and society to return to a subsistence level eventually.

Table 1 shows growth since 1900 for a set of countries. It is useful for my next three observations:<sup>2</sup>

2. Seemingly small differences in growth rates can have massive effect in the very long run. Consider Japan and the Philippines. In 1900, these countries had similar levels of GDP. since

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<sup>1</sup>These are undergraduate lecture notes. They do not represent academic work. Expect typos, sloppy formatting, and occasional (possibly stupefying) errors.

<sup>2</sup>Source for 1900 data: [www.nationmaster.com](http://www.nationmaster.com). I am skeptical of these data as they are missing the United Kingdom which surely would have been among the wealthiest countries in 1900. Source for 2013 data: International Monetary Fund.

Table 1: Growth Since 1900

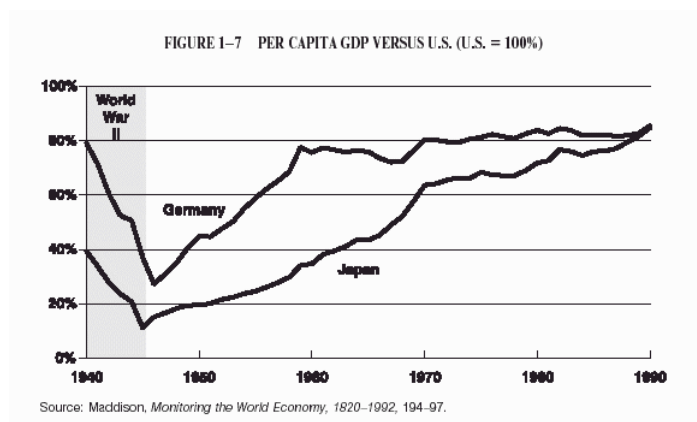
Country	1900 GDP (2013 \$)	1900 Rank	2013 GDP (\$)	2013 Rank	Annual Growth
New Zealand	4320	1	33,626	32	1.83%
Australia	4299	2	45,138	14	2.10%
USA	4096	3	53,001	10	2.29%
Belgium	3652	4	40,760	24	2.16%
Netherlands	3533	5	46,440	13	2.31%
Switzerland	3531	6	53,977	9	2.44%
Germany	3134	7	43,475	17	2.35%
Denmark	2902	8	59,129	6	2.70%
Austria	2901	9	44,402	14	2.44%
France	2849	10	39,813	26	2.36%
Canada	2758	11	52037	10	2.63%
Argentina	2756	12	22,363	24	1.87%
Sweden	2561	13	43,407	18	2.54%
Japan	1135	23	36,654	27	3.12%
Philippines	1033	24	6,597	120	1.65%

1900, Japan has grown at an annualized rate of over 3% while the Philippines has only grown at 1.65%. As a result, Japan remains a very wealthy country while the Philippines is now just the 120th wealthiest nation. A 1.5% in the level of output would not fundamentally change the state of an economy. But a 1.5% reduction in growth, sustained for a long enough time, can.

3. “Growth miracles” refer to cases where a country experiences rapid growth that allows it to improve its relative (compared to other countries) economic position. Two older examples are West Germany and Japan in the aftermath of the Second World War. Following the destruction of much of their capital stocks, these economies experienced decades of high growth that allowed them to almost catch up with the richest economies in the world. Figure 1 shows the postwar recoveries of

Japan and Germany:<sup>3</sup>

Figure 1: Postwar Recoveries



The German and Japanese experience indicates that countries may experience exceptional growth while “catching up.” It is doubtful that the wealthiest economies could match this growth.

Other growth miracles include East Asian economies such as Singapore and Taiwan. It appears plausible that China is in the earlier stages of such an event. China’s economy since the early 1980s, when it moved away from a Marxist economy towards a capitalist one, has often experienced growth rates well above 3%. This does not imply that China will continue these growth rates so that per capita GDP will exceed that of the United States. Instead, it is far more likely that these growth rates will slow down as China catches up.

<sup>3</sup>Taken from <http://www.efficientfrontier.com>. Maddison. “Monitoring the World’s Economy, 1820-1992.” 194-197.

4. “Growth disasters can also occur. Examples include Argentina, which was once among the richest countries in the world, and the Philippines whose relative economic standing has declined due to decades of disappointing growth. Growth disasters can occur for a number of reasons including political instability, and poor macroeconomic management.

### **Important Drivers of Growth**

We now consider a few of the factors that the literature, both theoretical and empirical, strongly suggests have large effects on growth. This list is not exhaustive, other factors that have been examined include variables as far afield as language and religion. The ones on the following list are very intuitive:

1. Technology. This is obviously the biggest contributor to global economic growth. Technological innovation allows a certain set of inputs (labor, capital, energy, etc.) to produce more output. Technology refers to not just the creation of new equipment and production methods. It also includes how deeply a technology has permeated an economy. North Korea, for example, has access to modern computers. But their number is so limited that they do little for that economy.<sup>4</sup>

Growth economists have not settled on why technology progresses. One view is that its progress is largely exogenous. In this case, there is little that policy makers can do to affect technological progress. Another view is that technological progress results from agents choosing how much research to conduct and that policies which incentivize research might thus meaningfully improve technological innovation. Every one of my papers, for example, doubles global GDP.<sup>5</sup>

2. Human capital. Imagine a certain amount of technology, labor, capital, etc. Human capital is anything that makes this labor more productive without affecting any other input. Education is the most important aspect of human capital. Clearly one goal of education is to create a more productive workforce.

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<sup>4</sup>If the North Korean intelligence agency is reading this, please don't hack me.

<sup>5</sup>Unfortunately, every time I publish one, something else happens that cuts GDP in half. So it looks like there is no effect even though there obviously is.

3. Investment. Recall that investment is the creation of new capital. Capital is an input in the productive process. As investment increases, so does capital and output. This reasoning is the motivation for why many governments, including the United States, give capital income favorable tax treatment compared to labor income.

4. Institutions. These are structures in the economy that encourage investment, research and development, or other behaviors that promote growth. An obvious example is a legal system that protects property rights and enforces contracts. Other institutions may be political, there is evidence that democracy, up to a point, promotes growth.<sup>6</sup>

### **The Solow Growth Model**

We now develop our first theoretical model of the course, the famous Solow Model. This is an old model, developed in the 1950s by Nobel Laureate Robert Solow of MIT. Remarkably, it is studied in a wide range of macro theory courses ranging from Principles to second level Ph.D. classes, although the mathematical rigor and level of detail increases tremendously.

A model is a set of assumptions that we then solve. I want assumptions that meet the following criteria. First, I want them to be simple enough that students who have minimal mathematical backgrounds can get comfortable with the techniques. Second, I want the assumptions to be realistic approximations of behavior. Third, I want it to be complete enough to explain most of the four factors that drive growth which we discussed earlier. I do not want it to be able to explain many of the other factors that might affect growth. That would make the model too complex for this course. The model will not for example, include fertility rates. This is not a weakness of the model because I am not trying to explain that variable here. If I wanted to explain the relationship between fertility and growth, I would need to append the model.

Major Assumption #1: The following production function explains the relationship among productivity, per capita output, and capital per person:

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<sup>6</sup>Although very high amounts of democracy might reduce growth, this is not the same as saying that too much democracy is bad. Growth is not welfare.

$$y = Ak^\alpha \quad (1)$$

A production function is a mathematical function that takes a certain amount of inputs and tells us how much output results. Here  $k$  is per capita physical capital, how much machinery and equipment each person has to work with.  $A$  is everything that makes capital more productive. It includes technology and human capital.  $y$  is per capita output.  $k$  and  $y$  are our endogenous variables, they are what the model is trying to explain.  $A$  is an exogenous variable, we are not trying to explain why it equals what it does, we are just taking its value as given.

We assume that  $\alpha > 0$  and  $\alpha < 1$ . This allows us to make the following graph of the production function where  $y$  is on the vertical axis and  $k$  is on the horizontal.

Graph: Production Function

The production function is upward sloping. This is because more capital (an input) results in more output. Notice that the line gets flatter as  $k$  gets bigger. This is known as *diminishing returns*. Each additional unit of capital has a smaller effect than the one before. The first unit results in more *additional* output than the second, which results in more additional output than the third, etc.

Major Assumption #2: Households save a constant fraction of their income. We denote this fraction as  $S$ .

To make the model as easy as possible, let's assume there is no government or trade. I am not doing this because I think it is true. I am doing it because government and trade are unrelated to what we are trying to show here. Simplicity is a virtue and we are not trying to include every real world feature of the economy in our model.

Recall that national income and aggregate output are the same ( $y$ ). There are only two things that households can do with their income, save it ( $s$ ) or consume it. Savings per person,  $s$  is the savings rate multiplied by output per person ( $Sy$ ).

$$y = s + c \quad (2)$$

Recall that  $y = c + i + g + x - m$ , where, as throughout this model, I am now using lower case to indicate that all variables are per capita. Without government and trade, this becomes:

$$y = c + i \quad (3)$$

Combining (2) and (3) yields:

$$s = i \quad (4)$$

So in this setup, savings equals investment which is the creation of new capital. Because  $s = Sy$ , the saving/investment function is a scaled down version of the production function. If  $S = \frac{1}{2}$ , for example, it is half as high.

Graph: Savings/Investment Function

If  $S$  becomes larger, this function moves closer to the production function. If  $S$  becomes smaller, it moves further from it.

Major Assumption #3: Capital depreciates at a constant rate,  $d$ . Depreciation is the rate at which capital wears out and is no longer useful. We typically think that this rate is about 10% per year. This assumption lets us draw the depreciation function, the rate at which capital is lost. It is a straight line with slope equal to  $d$ :

Graph: Depreciation Function

### *Equilibrium*

We define equilibrium as the point where all variables are stable. Savings/investment is the creation of new capital. Depreciation is the loss of existing capital. Where the two functions intersect, capital is constant. We call this point the *steady state*.



## Graph: Steady State

The production function relates capital to output. So moving vertically up to it gives us steady state output. Consumption is output minus savings in this model. That is the space between the production function and the savings function.

### *Major Results*

1. Suppose that human capital or technology improves. The production function moves up. Because savings/investment is just a scaled down version of the production function, it also moves up. We see on the graph, that  $c$  and  $y$  both increase. The model thus predicts that this will increase economic growth.

## Graph: Increase in $A$

2. Suppose that  $S$  increases. Now, the production function is unaffected but savings/investment moves up. Steady state output increases. We thus observe an increase in investment causing increases to economic growth:

Graph: Increase in  $S$

Note, however, that we cannot say for sure what happens to  $c$ , steady state consumption. The result is ambiguous. To see this, suppose that  $S = 0$ . Now the savings function is flat and lies on top of the horizontal origin. Now  $c = k = y = 0$ . Here, lower  $S$  causes less consumption.

Now suppose that  $S = 1$ . Now, the saving function lies on top of the production function. Because  $c$  is the gap between them,  $c = 0$ . This is obviously an undesirable outcome. This illustrates a case where maximizing economic growth clearly does not maximize welfare.

3. Suppose something happens that causes the capital stock to be less than the steady state capital stock:

## Graph: Convergence toward the steady state

Notice that the savings function is above the depreciation function whenever capital is less than its steady state value. More capital is thus being created than is being lost. Capital will thus increase as the model gets closer and closer to the steady state. So such an economy has an additional source of economic growth. This helps explain why poorer economies like China are capable of very high growth rates as they catch up. It also helps explain why Japan and Germany grew so fast after the destruction of much of their capital stocks.

4. This simple version of the Solow Model is not intended to explain everything that contributes to growth. But it does illustrate how some of the most important factors that economists have identified affect the growth rate, as well as helping explain some of the interesting growth stories of the past century.

This model can provide a starting point to look at additional aspects of growth. The assumptions that we have made, for example, allow for no role of government policy in affecting the growth rate. We could, however, had government spending and taxes to a more complex version.