Midterm B Key

1.) $r_t = p + \phi p^e_{t+1} - \phi p^* - p^e_{t+1} + p_t$

a)

$$p_t = (1-\phi)\, p^e_{t+1} + \hat{r}_t \quad , \hat{r}_t = r_t - p + \phi p^*$$

Iterating forward

$$p_t = (1-\phi)^j p^e_{t+j} + \hat{r}_t + \hat{r}^e_{t+1}(1-\phi) \dots + \hat{r}^e_{t+j}(1-\phi)^j$$

as $j \to \infty$ there is a unique solution if

$$|1-\phi| < 1 \quad , \quad \phi \in (-2, 0)$$

b.) $$p_{t+1} = \frac{p_t - \hat{r}_t}{1-\phi} + \delta_{t+1}$$

where $\delta_{t+1} = p_{t+1} - p^e_{t+1}$

c.) Recall $\pi_t = B E_t[\pi_{t+1}] + k \hat{y}_t$

$$k = \frac{(1-\theta)(1-B\theta)(1-\alpha)}{\theta(1-\alpha+\alpha\varepsilon)} \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$$

$$= \frac{(1-\theta)(1-B\theta)}{\theta(1+\alpha(\varepsilon-1))} \left((1-\alpha)\sigma + \varphi + \alpha\right)$$

$$\frac{dk}{d\alpha} = (1-\sigma)\left(\frac{(1-\theta)(1-B\theta)}{\theta(1+\alpha(\varepsilon-1))}\right)$$

$$- \frac{(1-\theta)(1-B\theta)(\varepsilon-1)}{\theta(1+\alpha(\varepsilon-1))^2}$$

sign depends on

$(1-\sigma) - \frac{\varepsilon-1}{1+\alpha(\varepsilon-1)}$ which is ambiguous

although if $\sigma > 1$ $\frac{dk}{d\alpha} < 0$

In this case, as $\alpha \uparrow$, $k \downarrow$ and

a given value of $\pi_t - \pi^e_{t+1} B$ yields a larger deviation

of $\hat{y}_t$.

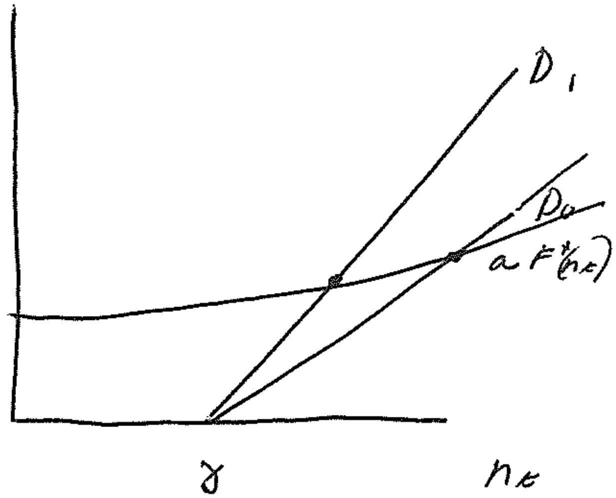
d.) Recall $Q_{t,t+k} = B^k \left(\frac{P_t}{P_{t+k}}\right)\left(\frac{C_t}{C_{t+k}}\right)^\sigma$

if $\frac{C_t}{C_{t+k}} > 1$ then $Q_{t,t+k}$ is higher than

its steady state Firms place more weight on future consumption because their marginal utility of consumption will be higher.

2)

a.)



D_1

~~More~~ Fewer sectors will be open and agents will invest a ~~greater~~ lesser amount in each open sector. There is ~~more~~ less risk taking because households are ~~better~~ less able to diversify.

b.) Because a ~~greater~~ lesser share of capital is invested in the assets with a higher expected return, convergence will occur ~~quicker~~ slower on average.

c.) ~~Recall in the 2 sector model~~ :

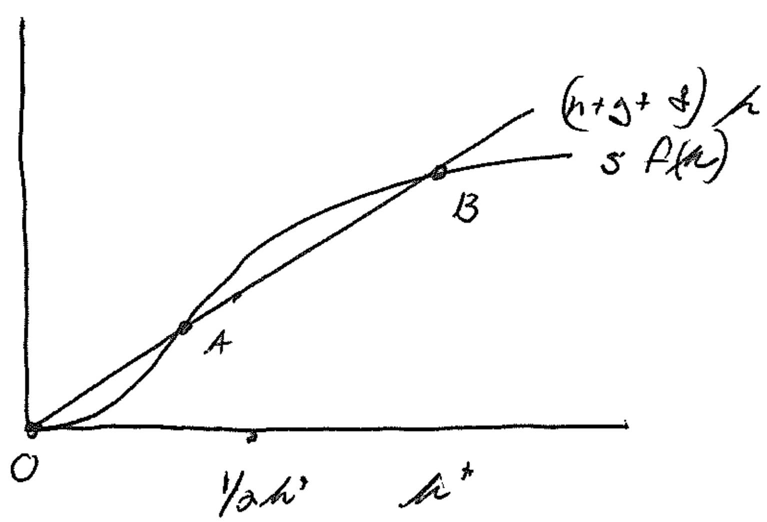
$r = \beta$

Yes. If $D = 0$, then all agents are always able to fully diversify. There is no more uncertainty in the model.

d.) False. The high and low growth steady states are only locally stable. They are learnable only if agents start in the basins of attraction.

3.

a.) there are many possibilities. One is that s starts as a convex function of k (low values) and then becomes concave for higher values of k.



O and B are stable steady states
A is unstable.

b.) In my example it all depends on whether or not $1/2 k^*$ is greater than the capital stock associated with A, the unstable steady state. If so, then the model will converge back to B. If not, it will converge to 0.

c.) Adaptive learning did start as a way to select among multiple equilibria. In this model, however, there are no expectations so it use makes little sense.

d.) Recall the Euler Equation:

$$\frac{C_{2t+1}}{C_{1t}} = \left(\frac{1+r_{t+1}}{1+\rho}\right)^{1/\theta}$$

As $\theta \to \infty$, $\frac{C_{2t+1}}{C_{1t}} \to 1$. and agents perfectly smooth their consumption without responding to changes to interest rates.

4.)

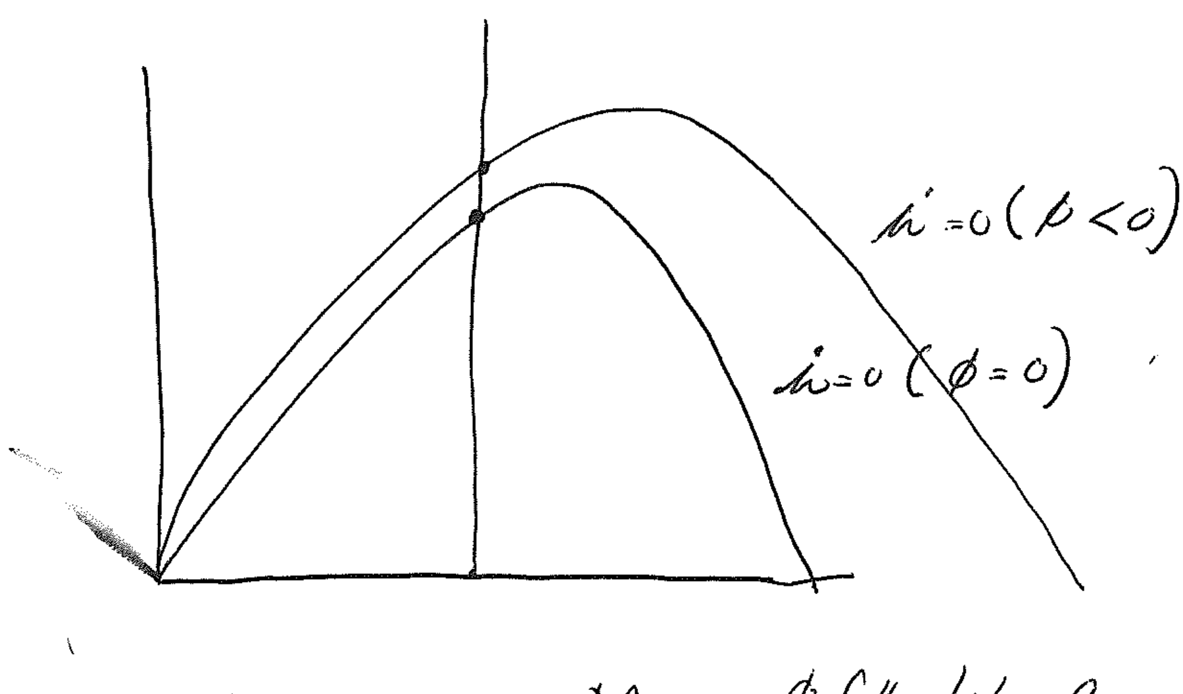
a) $\frac{d\ln(n)}{d\ln(c_t)} = \phi$, ϕ captures how changes to consumption impact fertility.

If $\phi > 0$, then wealthier households are more fertile.

b) I'll go with $\phi < 0$ as fertility rates are usually higher in lower income countries.

ϕ impacts the steady state through the capital accumulation equation.

$$\dot{k} = f(k(t)) - \cancel{(n+g)k} - c(t) - \left(c^{\phi}(t) + g\right)k(t)$$



k^* is unchanged. $c^* \uparrow$ as ϕ falls below 0.

c.) $$\mathcal{L} = B \int_{t=0}^{\infty} e^{-Bt} \left[\frac{C_t^{1-\theta}}{1-\theta} + V C_t^{\phi} \right] dt$$

$$- \lambda \left[h(0) - \int_{t=0}^{\infty} e^{-R(t)} e^{(C_t^{\phi}+g)t} \left(C(t) - w(t) \right) dt \right.$$

- we would then set $\frac{d\mathcal{L}}{d\, C_t} = 0$

d.) Yes. Markets are complete so equilibrium is efficient.