Midterm A Key

1.) a.)

$$r_t = p + \phi p_t + (1-\phi) p^e_{t+1} - p^* - p^e_{t+1} + p_t$$

$$p_t = \frac{\phi}{(1+\phi)} p^e_{t+1} + \frac{\hat{r}_t}{(1+\phi)}, \quad \hat{r}_t = r_t - p + p^*$$

Iterating forward

$$p_t = \left(\frac{\phi}{1+\phi}\right)^j p^e_{t+j} + \frac{\hat{r}_t}{1+\phi} + \frac{\hat{r}^e_{t+1}}{(1+\phi)^2} \cdots + \frac{\hat{r}^e_{t+j}}{(1+\phi)^{j+1}}$$

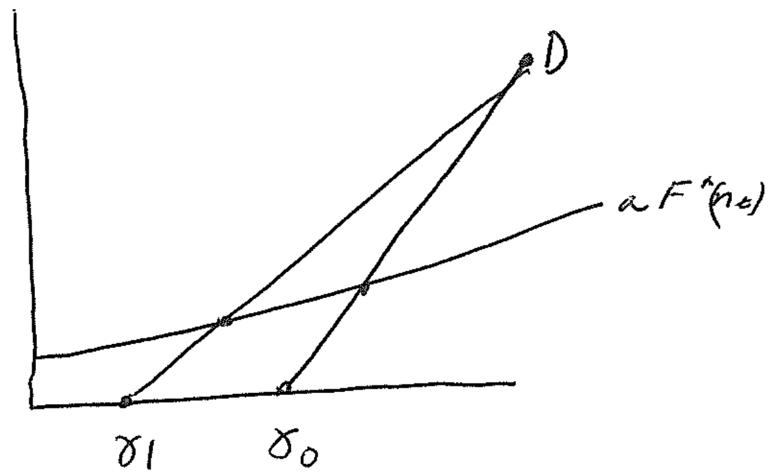
If $\left|\frac{\phi}{1+\phi}\right| < 1$, then as $j \to \infty$, the model converges to a unique solution.

c.) $-\sigma E_t[c_{t+1}] + \sigma c_t = \ln(B) + r_t$ when $x_t = \ln(X_t)$

d.) $\max_{p_t^*} \sum_{k=0}^{1} E_t[\,\vdots\,]$ Same as class.

b.) $p_{t+1} = \frac{1+\phi}{\phi} p_t - \frac{\hat{r}_t}{\phi} + \delta_{t+1}$ $\quad \delta_{t+1} = p_{t+1} - p^e_{t+1}$

2. a.) A lower value of γ means there are fewer projects without a minimum capital threshold.



$\gamma_1 < \gamma_0$

Fewer projects are open and agents invest less in each open project.

b.) Agents are investing more in the riskless project that yields a lower expected return. On average, convergence will take longer.

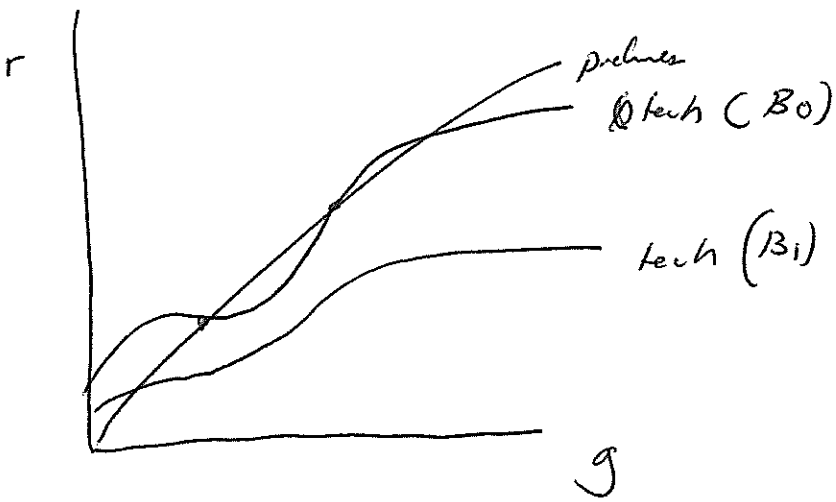
c.) Recall the 2 sector model

$$r = \frac{B}{x'(gk-1)} \qquad \frac{dr}{dgk} = \frac{B\, x''(gk-1)}{x'(gk-1)} < 0$$

the technology locus becomes steeper.

This change has many possible effects on the model. One possibility is that it eliminates the multiple equilibria.

$B_1 > B_0$



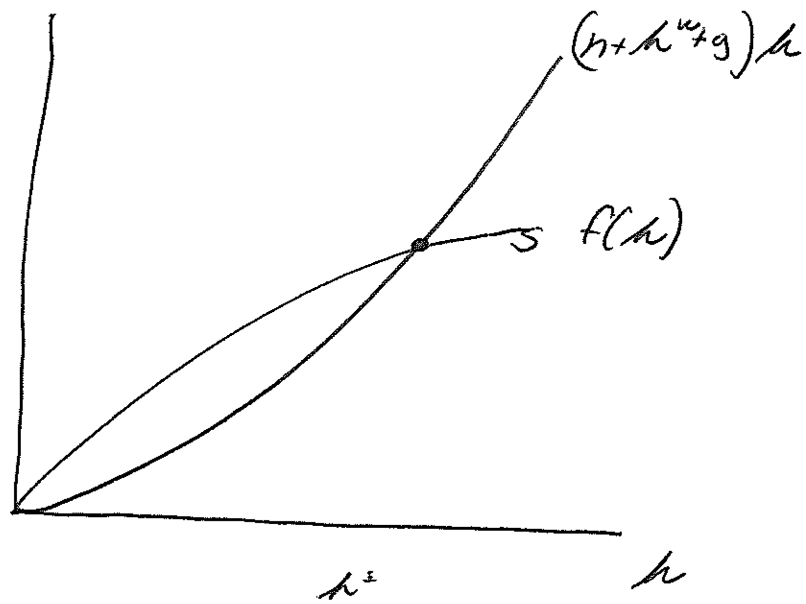
d.) I argue they do not. As seen in Evans, Honkapohja, and Romer (1998), as the sample size $\to \infty$, the high and low growth steady states are learned and the model acts just like perfect foresight (with the mid-growth steady state now considered uninteresting).

3.)

a.) $\dot{h}(t) = s\, f(h(t)) - (n + h^{w}(t) + \delta)\, h(t)$

b.) I will assume $w > 0$, although a reasonable answer can assume $w < 0$.

This assumption implies that the depreciation function becomes steeper as h increases.



It does not affect the stability of the unknown steady state

c.) Recall

$$h^* = \left(\frac{1-\alpha}{(1+n)(2+\rho)}\right)^{\frac{1}{1-\alpha}}$$

$$\frac{dh^*}{d\rho} = \frac{-1}{1-\alpha}\left(\frac{(1-\alpha)}{(1+n)(2+\rho)}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{1-\alpha}{1+n}\right)(2+\rho)^{-2} < 0$$

More patient households imply a lower value of ρ, $h^* \uparrow$

For c^*, set $g = n = 0$ without loss of genrlty.

$$k^* = \left(\frac{1-\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$$

$$k^{*\alpha} = c^* + k^*$$

$$c^* = \left(\frac{\delta+\rho}{1-\alpha}\right) \qquad \frac{dc^*}{d\rho} = \frac{1}{1-\alpha} > 0$$

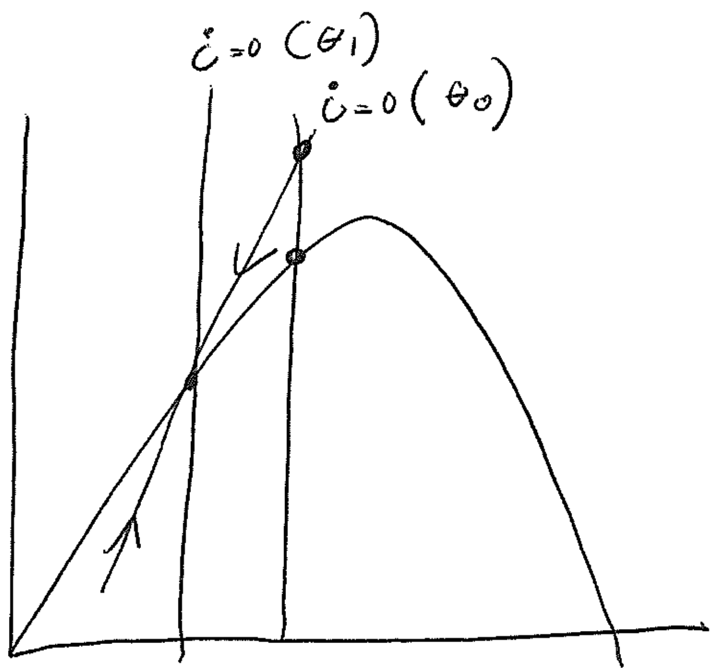
d.) In genrl, there are 2 effects on lowerg ρ on c. It increases k^* which we would expect to raise c^*. It also lowers $(1-s(r))$, which will lower c^*. Here, the former effect dominates.

4. a.) As $\theta \uparrow$, the intertemporal elasticity of substitution, $\frac{1}{\theta} \downarrow$. Households become less willing to shift their consumption in response to changes in the real interest rate.

b.) The only impact is on the Euler Equation

Recall $f'(\hat{k}) = \rho + \theta g$

As $\theta \uparrow$, $f'(\hat{k}) \uparrow$, $\hat{k}^* \downarrow$



$\theta_1 > \theta_0$

Here, c jumps up at the time of the change. It then falls to its lower steady state.

c.) Max $\frac{c_{1t}^{1-\theta}}{1-\theta} + \frac{B x^{1-\theta} c_{2t+1}^{1-\theta}}{1-\theta}$

Define $B' = B x^{1-\theta}$

$$\rightarrow \frac{c_{2,t+1}}{c_{1t}} = \left(\frac{x^{(1-\theta)}}{1+\rho}(1+r_{t+1})\right)^{1/\theta}$$

d.) We could just use the new, compound discount factor $B x^{1-\theta} = \frac{x^{1-\theta}}{1+\rho}$

Here, it is important to note that an OLG model becomes an infinite horizon model with altruism.