**A Macroeconomic Model with a Financial Sector**

These notes follow:

Brunnermeier, M. and Y. Sannikov. 2016. "A Macroeconomic Model with a Financial Sector." *American Economic Review*, Vol. 1(2): 379-421.

Iacoviello (2005) and related work shows that financial market frictions (modeled using a credit constraint) allows for macroeconomic shocks to be amplified and propagated. This then causes the economy to exhibit excess volatility relative to a model without financial frictions. Recall, however, that the quantitative magnitude of this amplification was fairly small.

This paper uses a model similar to Iacoviello (2005). A key difefrence, however, is that it does not rely on a log-linearization. It turns out that the non-linearities allow for the paper to make some important contributions.

1. The effects of shocks are highly non-linear. Shocks that keep the model somewhat close to the steady state do not produce dynamics that are particularly novel. Large, negative shocks, however, may push the economy into a crisis region where it may remain for an extended period of time. The model is thus both non-linear and asymmetric.

2. Risk is not just a function of the model's shocks, but is endogenous. In the model, if credit constraints might cause a drop in asset prices, agents may respond by selling assets for cash in anticipation of buying assets at lower prices. This behavior can then cause a drop in asset prices.

3. In the crisis region, the amplification of shocks is large and persistent.

4. Financial innovation, modeled as allowing agents to manage their risk, is often counter-productive because it increases systematic risk which then de-stabilizes the aggregate economy.

*Model*

There are two types of agents, experts and households. The key difference is that experts are more productive than households. Experts' production function is:

$$y_t = ak_t \tag{1}$$

Capital adjusts according to:

$$dk_t = (\Phi(\iota_t) - \delta)k_t dt + \sigma k_t dZ_t \tag{2}$$

Here, $\phi$ is an investment function chosen by the representative expert. It includes capital adjustment costs where $\Phi(0) = 0$, $\Phi^{'}(0) = 1$, $\Phi^{'}(\cdot) > 0$, and ,$\Phi^{''}(\cdot) < 0$. The term $dZ_t$ captures random shocks to capital.

Experts maximize the following:

$$E\left[\int_0^\infty e^{-\rho t} dc_t\right] \tag{3}$$

the notation here is unusual. $c_t$ is cumulative consumption so that $dc_t$ is the level of consumption.

Households face an analogous problem. In the paper, bars below parameters or variables indicate the values for households. There are three important differences:

1. Households discount at rate $r$ where $\rho > r$. Experts are thus less patient than households. They will thus borrow from households.

2. $\underline{a} < a$. Households are less productive than experts They may also face different depreciation rates.

3. Experts cannot have negative consumption (a typical assumption). Households, however, can. Footnote 9 attempts to justify this unusual assumption by interpreting negative consumption as a disutility from having to supply additional labor.

Unusually, both types of agents are risk neutral.

The authors now put some restrictions of experts. Experts cannot sell equity in their firms. The only asset that they may sell to households is risk-free debt. This is dome to prevent experts from selling all their equity to households and then consuming all of their wealth immediately. They would otherwise wish to do so because they are risk-neutral.

*The Expert's Problem*

Experts must choose how much to consume ($dc_t$), and how much of its wealth to hold as capital ($x_t$), and how much to hold as risk free debt. Their net worth thus evolves according to:

$$\frac{dn_t}{n_t} = x_t dr_t^k + (1 - x_t)rdt - \frac{dc_t}{n_t} \tag{4}$$

Experts' problem then consists of maximizing (3) subject to (4). Households face an analogous problem.

In equilibrium, experts use leverage. This is indicated by $x_t > 1$ showing that experts borrow from households at the risk free rate in order to purchase additional capital. Intuitively, there is a market for this type of financial transaction because capital is more productive in the hand's of experts than households.

Experts must hold non-negative net worth at all times. As a result, they have a precautionary motive where they hold additional capital (and thus forego consumption) in order to product against a negative draw of $Z_t$ which implies that capital is less productive.

Even though they are risk-neutral, experts act as if they were risk-averse. This is because negative shocks suppress asset prices and thus offer attractive investment opportunities. By taking on more risk, however, they make themselves less able to exploit such opportunities because their own net worth is lower when negative shocks occur.

The authors define $\eta_t$ as experts' share of the economy's wealth. They show that the price of capital ($q_t$), the share of capital held by experts ($\psi_t$), and the maximized value of expected consumption ($\theta_t$) all depend only on $\eta_t$. Figure 1 shows these functions for a calibrated version of the model. Solving the model thus consists of solving for $]eta_t$ for a history of productivity shocks, and then using these functions to solve for the other variables of interest.

The price of capital is an approximately linear function where $q_t$ increases with $\eta_t$. This is intuitive because capital is more expensive when more of it is held by experts who are more productive.

Experts' share of capital rises with $\eta_t$ until they own all capital around $\eta_t = 0.2$.

*Dynamics Around a Steady State*

The authors define $n^*$ as the non-stochastic steady state. This is a point where the model would converge to if shocks always equaled zero. Notably, the authors are not log-linearizing around this point. Instead, they want to see the model's non-linearities which would be lost were they to use a linearization.

Importantly, equilibrium near this steady-state is not efficient. There are three sources of inefficiency:

1. $\eta < 1$ is an inefficiency because experts would ideally manage all capital (due to their higher productivity).

2. Because capital is less productive, its price is lower. This causes experts to underinvest in capital.

3. Because they are risk neutral, experts would like to consume all of their consumption right away. Financial market frictions, however, compel them to postpone consumption.

The authors make a distinction between two types of risk. The first is fundamental risk which is exogenous and depends on the productivity shocks, $Z_t$. The second is endogenous risk (denoted $\sigma_t^q$) and exists because of financial frictions. Around the steady state, endogenous risk is low. If the model is well below the steady state, however, experts fear that they may be unable to maintain positive net worth in the future and endogenous risk can become large. This may then trigger an adverse feedback loop (shown in Figure 3) that traps the economy for an extended period f time well below the steady state. The process works as follows:

1. An adverse productivity shock occurs.

2. all agent respond by reducing investment and capital. Because they are leveraged, experts' share of wealth ($\eta_t$) falls.

3. As experts own less capital, more of it is owned by less productive households. This causes its average marginal product and price to fall.

4. Because they are leveraged, this causes a further drop in $\eta_t$ which takes the model back to #2.

5. Because there is less capital and capital is less effectively allocated, output also falls.

The right panel of Figure 4 shows the $\eta_t$. This illustrates how frequently the economy achieves each value of $\eta_t$. Notably, the distribution is bi-modal. The global peak occurs near the steady state where experts own most of the capital. But there is another peak where households own most of the capital. This shows hoe the model can occasionally be stuck in a low productivity/output regime following a series of adverse shocks.

*Other Results*

1. The authors identify a volatility paradox. We might expect that as the standard deviation ($\sigma$) of the model's only fundamental shock approaches zero, then the economy ceases moving into the crisis region. Table 1, however, shows that this is not the case by simulating several different values of $\sigma$. Consider a decrease in $\sigma$.

a. Because the risk of a crisis is initially reduced, experts respond by increasing their leverage. This increases the systematic risk borne by the economy.

b. As a result, smaller shocks are needed, and households need acquire a smaller share of capital, to induce a crisis. The space between the two modes is smaller.

c. Crisis thus occur more often and aggregate volatility is not substantially reduced.

2. The authors then modify the capital accumulation equation to include random borrowing costs:

$$dk_t = (\Phi(\iota_t) - \delta)k_t dt + \sigma k_t dZ_t + k_t dJ_t^i \tag{5}$$

where $dJ_t^i$ are the borrowing costs that are paid to households in exchange for their having to verify that experts hold positive net worth.

Borrowing costs are an inefficiency. usually, we expect that adding an inefficiency.will make the model's equilibrium less attractive. But here, the borrowing costs help reduce the impact of another inefficiency, financial frictions, and the economy is actually stabilized. Because they have to take on borrowing costs, experts are less leveraged in this version of the model. because there is then less systematic risk, crises happen less frequently.

3. The authors next consider idiosyncratic shocks where different lenders receive different draws of $dJ_t^i$ but common values of $Z_t$. They authors interpret financial innovation as allowing these agents to pool their risk. The authors reference financial instruments such as mortgage backed securities and credit default swaps as real world examples of this type of financial innovation.

Here, financial innovation moves the model back toward the version without borrowing costs. As a result, although they reduce individual risk, they incentive greater leverage, and thus increase systematic risk. Crises now occur more often.

4. The authors seek to identify policies that can improve welfare. The efficient allocation (where experts control capital) can only be achieved if policy makers aggressively creating and managing an insurance asset. Alternatively, welfare can be improved (but not be maximized) with the more limited policy intervention by creating an insurance instrument that protects experts against left tail risk. Such a policy can prevent experts from selling off all their capital when this risk is realized.