Linear Algebra Practice Key

a.)
$$\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 1 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 43 & 23 \\ 16 & 13 \end{bmatrix}$$

(2×3) (3×2) (2×2)

b.) Matrices are non-conformable

c.)
$$\begin{bmatrix} 3 & 9 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 43 & 23 \\ 16 & 13 \end{bmatrix} = \begin{bmatrix} 46 & 32 \\ 17 & 17 \end{bmatrix}$$

d) I said BC does not exist.

e.)
$$\begin{bmatrix} 3 & 9 \\ 1 & 4 \end{bmatrix}^{-1} = \frac{1}{12-9} \begin{bmatrix} 4 & -9 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{4}{3} & -3 \\ 1/3 & 1 \end{bmatrix}$$

f.) Det $(A) = 12 - 9$

g) C is not square and cannot be inverted.

h.)

$$\text{Det}\begin{bmatrix} 43-\lambda & 23 \\ 16 & 13-\lambda \end{bmatrix} = 0$$

$$(43-\lambda)(13-\lambda) - 16 \times 23 = 0$$

$$\lambda^2 - 56\lambda - 368 = 0$$

$$\lambda = \frac{56 \pm \sqrt{56^2 - 4 \times 368}}{2} = 50.7,\ 5.3$$

Eigenvectors

$$\begin{bmatrix} 43-50.7 & 23 \\ 16 & 13-50.7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-7.7x_1 + 23x_2 = 0 \qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2.99 \end{bmatrix}$$

$$\begin{bmatrix} 43-5.3 & 23 \\ 16 & 13-5.3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$37.7x_1 + 23x_2 = 0 \qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -.61 \end{bmatrix}$$

i) C is not square

j) B is not square

2) a.) $$\begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} e_t \\ u_t \end{bmatrix}$$

b.)

$$\begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -\alpha \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} \begin{bmatrix} 1 & -\alpha \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_t \\ u_t \end{bmatrix}$$

c.) $\lambda = 1, 1$

$$\begin{bmatrix} 0 & -\alpha \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$ for both eigenvectors.