HW #5 Key

1.) Denote $g(t) = \frac{G(t)}{A(t)}$

$$U = \frac{L(0)(1+\gamma)\ln A(0)}{H} \int e^{-\rho t} e^{nt} e^{gt} \left[\ln(c(t)) + \gamma \ln(g(t))\right] dt$$

2) The only difference is the tax rate

$$\int_{t=0}^{\infty} e^{-R(t)} e^{(n+g)t} \left(c(t) - (1-z)\, w(t)\right) \le k(0)$$

3.) $$\mathcal{L} = B \int_{t=0}^{\infty} e^{-\beta t} \left[\ln(c(t)) + \gamma \ln(g(t))\right] dt$$

$$-\lambda \left[k(0) - \int_{t=0}^{\infty} e^{-R(t)} e^{(n+g)t} \left(c(t) - (1-z)\, w(t)\right) dt\right]$$

where $\beta = \rho - n - g$ (not the same as class)

4. $$\frac{d\mathcal{L}}{d\,c(t)} = 0 = \frac{B e^{-\beta t}}{c(t)} - \lambda e^{-R(t)} e^{(n+g)t}$$

take logs

$$\ln(B) - \beta t - \ln(c(t)) = \ln\lambda - R(t) + (n+g)t$$

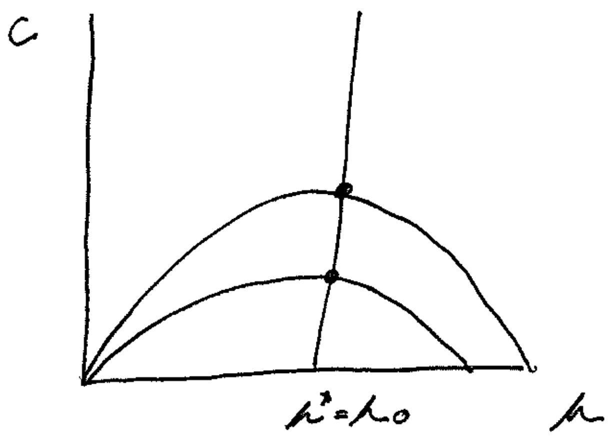
8 differentiate w.r. to t.

$$-\beta - \frac{\dot{c}(t)}{c(t)} = -r(t) + (n+g)$$

Recall $r(t) = f'k(t)$

$$\frac{\dot{c}(t)}{c(t)} = f'(k(t)) - \rho - g$$

5.) As z increases, breakeven inflation falls:



The model immediately jumps to its new steady state. Capital is unchanged, consumption is lower.

6.) I don't want to write the awful word.

Bonus:

$$\mathcal{L} = B \int_{t=0}^{\infty} e^{-\beta t} \left[\ln(c(t)) + \gamma \ln(z(t)) + \gamma \ln w(t) \right] dt$$

$$-\lambda \left[k(0) - \int_{t=0}^{\infty} e^{-R(t)} e^{(n+g)t} \left(c(t) - (1 - z(t)) w(t) \right) dt \right]$$

Note: optimization w.r to $c(t)$ is unaffected. Therefore the Euler Equation and steady state capital stock are unaffected

$$f'(k^*) = \rho + g$$

differentiate w.r to $z(t)$

$$\frac{B e^{-\beta t} \gamma}{z(t)} = \lambda e^{-R(t)} e^{(n+g)t} w(t)$$

$\Rightarrow$

$$-\beta - \frac{\dot{z}(t)}{z(t)} = -r(t) + (n+g) + \frac{\dot{w}(t)}{w(t)} \quad \Rightarrow \text{ just confirms } f'(k^*) = \rho + g$$

$$\frac{B e^{-\beta t} e^{R(t)}}{\lambda e^{(n+g)t}} = \frac{z(t) w(t)}{\gamma} = c(t) \quad \left[\frac{c^*}{z^*} = \frac{f(k^*) - f'(k^*) k^*}{\gamma} \right] \star$$

- when γ increases, the model's steady state capital stock is unchanged. From $\star$, however, the steady state consumption level falls and steady state z (and g) increase. The economy immediately moves to its new steady state.