ECO 318, Winter 2017, HW#1

Instructions: Answer all questions. This assignment is due at the start of class on 1/26.

1. Consider the following macroeconomic model of output (y_t) and stock prices (s_t).

$$s_t + \alpha y_t = y_{t+1} \tag{1}$$

$$s_t = .1s_{t-1} - .2y_{t-1} \tag{2}$$

a. Represent this model in matrix form.

b. Is this system stationary or explosive?

c. Does this system oscillate?

2. Consider the bourbon bottle problem. A person obtains utility both by drinking from a bottle of bourbon (d_t) and by displaying the remaining bourbon in a stylish bottle (b_t). In each period, their instantaneous utility is:

$$u(b_t, d_t) = ln(d_t) + \chi ln(b_t) \tag{3}$$

Bourbon evolves according to:

$$b_{t+1} = b_t - d_t \tag{4}$$

Furthermore, households discount as usual using the discount factor β.

a. Interpret the parameter χ.

b. Which variable is the control and which variable is the state?

c. Write out the value function so that it depends on instantaneous utility, (3), and the discounted value function for the next period.

d. Use (4) to eliminate the control variable and re-write the value function as only a function of the current and future state variable.

e. Make a guess at the functional form of the value function and insert this into your future value function on the right hand side of your answer from d.

f. Differentiate the value function with regard to the future state variable (which is also a control). Set this derivative equal to zero in order to obtain the future state variable as a function of the current state.

g. Verify your guess. Did you guess correctly? If not, try again and explain how you are improving your guess.

h. Solve for the coefficient(s) from your guess.

i. Use (4) to solve for the control variable as a function of the current state variable.

j. How does χ affect the optimal rate of bourbon consumption? Provide intuition.

k. What happens to bourbon consumption over time?