Growth Cycles[1]

These notes follow the paper *Growth Cycles* by Evans, Honkapohja, and Romer.[2] It represents a modern extension of the growth models that we have examined.

Recall Endogenous Growth

All students are familiar with Paul Romer's basic endogenous growth framework. In that setting, firms make a choice of how they allocate resources between production and research and development. TFP growth is increasing in the latter and firms choose to invest in an inefficiently low level of R&D because they do not care about the positive externality.

In this setting, two countries with different exogenous parameters generally have two different steady state growth rates. This is in contrast to neo-classical growth models where the same two countries would have different steady state levels of output, but their steady state growth rates would be the same (and largely a function of exogenous TFP growth).

This paper goes one step further. It modifies the Romer model in order too allow two countries with identical parameters to have different growth rates. Specifically, there are multiple stable steady states. All of you have seen an example of this in ECO 270 with the poverty trap extension of the Solow Model. That example, while important, left one major question unanswered: how would two otherwise identical countries end up at different steady states. this paper offers an explanation. It assumes that expectations matter and that agents form these expectations using standard econometric approaches, known as *adaptive learning*. Under learning, the model can randomly shift between the stable states.

Baseline Model (No Learning)

The authors start with a simple, discrete time, version of their model which they then append to generate their major results:

Households

[1]These are undergraduate lecture notes. They do not represent academic work. Expect typos, sloppy formatting, and occasional (possibly stupefying) errors.

[2]Evans, G., Honkapohja, S. and P. Romer. 1998. "Growth Cycles." *American Economic Review*, Vol. 88(3):495-515.

The representative household is assumed to behave in the standard way. Its instantaneous utility function is CES:

$$U(C) = \frac{C^{1-\sigma}}{1-\sigma} \tag{1}$$

and it discounts using the discount factor β: This yields a standard Euler Equation:

$$g_c = \frac{C_{t+1}}{C_t} = [\beta(1+r)]^{\frac{1}{\sigma}} \tag{2}$$

where g_c is defined as the growth rate of consumption, the key variable throughout the paper.

Firms

Initially assume that production is linear:

$$Y = BK_t \tag{3}$$

and

$$K_{t+1} - K_t = Y_t - C_t \tag{4}$$

Note that there is no capital depreciation.

It is more interesting to assume that there are two sectors in the economy. This represents an important, though not original, assumption of the authors. Replace (4) with:

$$C_t = Y_t - K_t \chi \left(\frac{K_{t+1} - K_t}{K_t} \right) \tag{5}$$

The key feature is a capital adjustment cost. We assume that the function χ is convex and that $\chi(0) = 0$. This assumption introduces a degree of imperfect substitutibility between the production of capital goods and consumption goods. It is no longer the case that one unit of output can either be consumed or turned into capital. Instead, it can be turned into capital with decreasing effectiveness.

Consider the price of capital in terms of the consumption good. This is simply the derivative of C_t with respect to K_t from (5), which equals $\chi'(g_k - 1)$ where $_k$ is the growth rate of capital. Suppose a household wishes to save/invest one unit of the consumption good. It has two choices:

1. It can purchase $1/\chi'(g_k - 1)$ units of the consumption good. using (3), this will then yield

a flow of B units of the consumption good.consumption goods. So the household is then left with $\frac{B}{\chi'(g_k - 1)}$ of consumption goods.

2. It can simply save that unit at the risk free rate r which then yields r units of consumption each period.

Arbitrage suggests that these two options must yield the same return:

$$r = \frac{B}{\chi'(g_k - 1)} \tag{6}$$

Before moving on, note that (4) implies that g_k=g_c. We will hereafter denote both as g.

Figure 1 in the paper shows how this initial version of the model may be solved using a simple graph. Equation (2) posits a simple upward sloping relationship between g and r. Equations (3) and (4) suggests that r, capital's marginal product, is constant in the one technology version of the model. Now examine (6). As g increases, χ' increases because χ is convex. This implies a negative slope.

The steady state growth rate is just determined by the intersections of these two relationship. In both cases, it is unique. So far, the model is not yielding any unusual results.

Complementary capital goods

To get interesting results, we start by making the unobjectionable assumption that there are actually many different types of capital goods. Redefine the production function so that it equals:

$$F(L, x(\dot{})) = L^{1-\alpha} \int_0^A x(i)^\alpha di \tag{7}$$

Note that A makes an appearance. It is now the number of capital goods in existence. We wills see the connection with TFP soon.

We now make an additional assumption based of of the Endogenous Growth literature, it takes a units of forgone consumption to invent a new type of machine. Thus:

$$Y_t = C_t + a(A_{t+1} - A_t) + K_{t+1} - K_t \tag{8}$$

where $K_t = \int_0^A x(i)^\alpha di$.

So far, this is a lot of work to get very little. The graph still is the same as the on technology case in Figure 1. For interesting things to occur, we now make one of the model's core assumptions, that different types of capital goods are compliments:

$$F(L, x(\dot{})) = L^{1-\alpha} \left(\int_0^A x(i)^\gamma \right)^\phi di \tag{9}$$

If $\phi > 1$, then the capital goods are compliments. Intuitively, the invention of the computer increases the benefit to inventing a new operating system. For technical reasons, the authors must assume that capital goods become harder to invent as technology progresses. Specifically, it takes ι^ξ units of forgone consumption to invent the i^{th} capital good where:

$$\xi = \frac{\phi - 1}{1 - \alpha} \tag{10}$$

We can then write the technology equation as:

$$Y_t = C_t + Z_{t+1} - Z_t \tag{11}$$

where

$$Z_t = \int_0^A x(i)^\alpha di + \int_0^A \iota^\xi di = K_t + \int_0^A \iota^\xi di \tag{12}$$

The key feature here (the math is shown in the appendix) is that the technology locus is now upward sloping. Recall that, without capital adjustment costs, it was flat. Now, however, increasing growth increases the opportunities for different types of capital to compliment each other. As growth increases, there are more compliments increasing each type of capital's marginal product of labor. For certain calibrations, the technology locus is convex resulting in Figure 2.

There are now two steady states. In principle, two identical (in terms of parameters) economies could find themselves with not just different levels of consumption, but different growth rates as well. The obvious question is which of these steady states are stable?

Recall that we did a related stability analysis with the Solow Model. In that case, we started away from the steady state capital stock and examine what happened in subsequent periods. There is no equivalent framework here. Instead we will conduct a stability analysis based on expectations.

So far, we have assumed *perfect foresight*. This means that agents know the correct value of all future and current variables, hence the lack of expectations operators. Perfect foresight is common in growth models. The authors, however, instead assume that the interest rate is not known. Households form expectations of the interest rate using the following adaptive learning rule:

$$r^e_{t+1} = r^e_t + \delta_t(r_t - r^e_t) \tag{13}$$

The parameter δ_t is the "gain." It represents the weight on the most recent expectational error. The authors set $\delta_t = t^{-1}$, the inverse of the sample size. The question is then does r^e_t converge to one or both of the steady states. They find that the model always moves toward the low growth steady state.

Multiple Stable Steady States

We started with a version of the model where the technology locus is flat. One modification, assuming two technologies through a a capital adjustment cost, made it downward sloping. Another, complementary types of capital, caused it to be upward sloping. Combining these yields a hybrid with 2 stable steady states.

The authors now use the following technology equation:

$$C_t = Y_t - Z_t\chi\left(\frac{Z_{t+1} - Z_t - D_t}{Z_t}\right) \tag{14}$$

The term D_t includes depreciation. I suspect a referee made them add this as it does not do much.

For some parameters, the model may exhibit the following properties:

1. The technology locus is initially above the preferences locus.

2. If growth is low, few new inventions occur. This limits the impact of complimentary capital. The technology locus is downward sloping due to the effects of capital adjustment costs.Where they cross is a stable steady state.

3. As growth increases, complimentary capital becomes very important causing the technology locus to be upward sloping. This results in an unstable steady state.

4. For growth rates too high, it becomes very costly to invent new types of capital. As a result, complimentary is again less important. The locus becomes downward sloping resulting in a second stable steady state.

The authors now calibrate their model by choosing numerical values for each parameter. They set $\alpha = 0.4$, $d = 0.15$, $\beta = 0.962$, $\sigma = 0.21$ and provide a specific functional form for χ.

This calibration yields two steady states. The low growth steady state corresponds to 0.2% growth and $r = 4\%$. The high growth steady state corresponds to 4.9% growth and $r = 5\%$.

Expectational Indeterminacy

Indeterminacy is a type of multiple equilibria where agent's extraneous beliefs may become self-fulfilling. Extraneous optimism causes the economy to do well while extraneous pessimism causes it to do badly. Indeterminacy is an old concept, Keynes described it when referring to the "animal spirits" of investors that drove investment and business cycles. Indeterminacy is a type of *speculative bubble*. Despite its appeal, macroeconomics has had a hard time finding circumstances where plausible assumptions result in its existence. This is an exception.

The authors employ a modeling device known as a "sunspot," denoted s_t. A sunspot is extraneous noise that is unrelated to anything fundamental (*e.g.* productivity). Intuitively, it should not matter, They key, however, is that if agents form their expectations using the sunspot, then there is some mechanism (like multiple steady states) that will allow their expectations to be correct and hence rational.

s_t formally follows a *2 state Markov switching process*. This means thats_t may equal either 1 or 2. If it equals 1 in period t then there is some exogenous probability that it will remain equal to 1 in period $t = 1$, and 1 less that probability that it will switch to equal 2 in period $t = 1$. the same applies if it equals 2 in period t, there is an exogenous probability that it will remain equal to 2 in the next period.

The authors now employ the following algorithm for adaptive learning:

$$r^e_{i,t} = r^e_{i,t} + \frac{\delta}{N_i(t)}(r_t - r^e_{i,t}) \;\; if\; s_t = i \tag{15}$$

$$r^e_{i,t+1} = r^e_{i,t} \;\; if\; s_t \neq i \tag{16}$$

where $N_i(t)$ is the number of periods that s_t has equaled $i = 1, 2$.

In words, households are forming two separate expectations, one for each possible value of s_t. If $s_t = 1$, they update their beliefs about the expected interest rate for that value of s_t while keeping their expectations for if $s_t = 2$ unchanged.

Proposition 1 (page 505) shows the next main result. If the starting values for $r^e_{1,t}$ and $r^e_{2,t}$ are close enough to the high and low growth stable steady states, then the learning process will converge. The state of the model (whether growth is low or high) will then depend on the ransom variable s_t.

This paper blurs the distinction between growth and business cycles. We may interpret y as potential output, as we typically do in growth models, and then model deviations from potential output separately. Or we may think of recessions as transitions to the low growth state.

Recap

This paper was published in the profession's top journal because it showed that plausible assumptions (complementary capital, learning, etc.) can result in important, interesting, and unexpected behavior. It presents an entirely novel explanation for volatile growth rates. It is also well executed.

The biggest limitation of this paper is that its results rely on specific parameter values and functional forms. It is an open question how robust its main findings are:

1. The authors employ a reasonable paramterization. But among the many other reasonable calibrations, how often does this result hold.?

2. How would the addition of other reasonable modeling assumptions affect the results? Would they be more or less general?

We leave these as open questions. This paper is purely theoretical. It may, however, cause one to think about testable implications. For example, it predicts that growth rates result from self-fulfilling beliefs about the return on capital. How might one measure these beliefs? What econometric specification might allow us to identify their effects on growth?

More on Multiple Equilibria

This paper demonstrates two types of multiple equilibria, multiple steady states and expectational indeterminacy. Both are of great interest. A few points about them:

1. Multiple equilibria almost always require some deviation from complete and competitive markets. In this paper, those deviations include production spillovers and adaptive learning (there is no market for information).

2. Multiple equilibria can occur in models with complete and competitive markets but there can be no welfare implications. The most obvious example is if $U(C) = C$.

3. In the business cycle literature, there has been great interest in determining the plausibility of indeterminacy as an explanation for business cycles. A few approaches have been taken:

a. Increasing returns to scale. It is easy to bring animal spirits by assuming that the aggregate production function exhibits increasing returns to scale. But this modeling approach has struggled to require plausible degrees of increasing returns to scale without having to add other problematic assumptions. Note that the authors of the presents papers make it clear that they do not rely on this approach.

b. A balanced budget where taxes automatically adjust to equal government spending. Interesting but implausible.

c. It also occurs in a New Keynesian Model where the monetary authority does not respond aggressively to inflation. This has been offered as an explanation for the volatility of the 1970s. Indeterminacy adds randomness to an economic system and usually worsens welfare.

d. The approaches from $a - c$ have not held if agents form expectations using adaptive learning.