The Basic New Keynesian Model

These notes follow parts of Gali Ch.3. They develop the basic New Keynesian Model. This framework, first developed in the 1990s, has emerged as the dominant framework for the analysis of business cycles, especially those that correlate with monetary policy. Mathematically, this model is very challenging to derive. By studying it we:

1. Understand the theory which motivates the Federal Reserve and other Central Banks in conducting monetary policy. Thus even if you think it is nonsense, it is still a useful model to understand because most policy makers like this model.

2. This is just a basic version of the model. There are limitless extensions. Suppose, for example, that we wish to examine optimal fiscal policy. By understanding the basic model which simplifies away fiscal policy, we have a framework that allows us to (fairly) easily add it.

3. The model isolates the assumptions needed to allow monetary policy to be non-neutral. The model includes two distortions from complete and competitive markets. First, firms operate under monopolistic competition instead of perfect competition. Second, prices are sticky. The second is what allows money to matter.

Households

Household optimization is similar to Ch.2. The one exception is that C_t is an index of varied goods instead of a single consumption good.

$$C_t = \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{1}$$

the parameter ϵ represents the degree of substitutbility across different goods. As $\epsilon \to \infty$, goods become perfect substitutes and the consumption index is simply their sum. As $\epsilon \to 1$ from above, the index approaches the minimum of the individual goods.

The representative household solves the following problem:

$$Max_{C_t} \;\; E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t) \tag{2}$$

s.t.

$$\int_0^1 P_t(i)C_t(i)di + Q_tB_t \leq B_{t-1} + W_tN_t + T_t \tag{3}$$

There are a pair of differences in the budget constraint versus Ch. 2. First, we must integrate over the index i to obtain expenditures on consumption. Second, we are ignoring money and instead simply assuming that the monetary authority can control i_t. Note that we can always add it back in by re-doing the money demand equation from Ch. 2.

Optimization yields the same Euler Equation and labor supply rule as Chapter 3. In addition:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \tag{4}$$

where $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon}di\right)^{\frac{1}{1-\epsilon}}$.

Formally obtaining (4) is a bit of work. This condition determines, within the index C_t., how much of each individual consumption good, $C_t(i)$, the household chooses.

First define $\int_0^1 P_t(i)C_t(i)di = Z_t$, the level of expenditures on consumption. Recall that C_t is determined by the Euler Equation. For any optimal value of Z_t, we can set up the following Lagrangian:

$$\mathcal{L} = \left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}}di\right]^{\frac{\epsilon}{\epsilon-1}} - \lambda\left(\int_0^1 P_t(i)C_t(i)di - Z_t\right) \tag{5}$$

Differentiating w.r.t. $C_t(i)$ yields:

$$C_t(i)^{\frac{-1}{\epsilon}}C_t^{\frac{1}{\epsilon}} = \lambda P_t(i) \tag{6}$$

Note that (6) is true for any $i \in [0,1]$. Thus for any two goods, i and j, it must be true that:

$$C_t(i) = C_t(j)\left(\frac{P_t(i)}{P_t(j)}\right)^{-\epsilon} \tag{7}$$

We can then use (7) to rewrite the definition of consumption expenditures:

$$\int_0^1 P_t(i)^{1-\epsilon}C_t(j)P_t(j)^{\epsilon}di = Z_t \tag{8}$$

Recalling that $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$:

$$P_t^{1-\epsilon} C_t(j) P_t(j)^{\epsilon} di = Z_t \tag{9}$$

Re-arranging (9) and switching from j back to i for notational convenience:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} \frac{Z_t}{P_t} \tag{10}$$

Now recall that $C_t = \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$, which can be re-stated using (10):

$$C_t = \left(P_t(i)^{1-\epsilon} di\right)^{\frac{\epsilon}{\epsilon-1}} P_t^{\epsilon-1} Z_t = P_t^{-1} Z_t \tag{11}$$

Equations (10) and (11) then combine to return (4):

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \tag{12}$$

Firm Optimization

Firms are indexed using i, each producing a differentiated good. All firms, however, utilize a common technology:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{13}$$

We have already introduced one distortion, monopolistic competition. We now add a second, known as Calvo (1983) pricing. This is an example of a nominal rigidity that makes a model Keynesian. We assume that each period, each firm is able to choose its price with probability $1 - \theta$. If not visited by the Calvo fairy, the firm is stuck with its price from the previous period (some homework questions allow the firm to instead index its price to some measure of inflation). Here are some thoughts on Calvo pricing:

1. Calvo pricing significantly complicates the firm's profit maximization problem. The firm must now choose its price knowing that it may be stuck with it for many periods into the future.

2. This represents the New Keynesians' efforts to establish microfoundations on old Keynesian concepts. Critics, however, argue that it does not represent much of an improvement. Their argument is based on the idea that there is no explicit reason why firms do not choose to re-optimize. Whether the model has good microfoundations is thus not obvious.

3. This is not the only way to add nominal rigidities. Other approaches include assuming sticky information (firms only update their information sets every few periods), sticky wages, overlapping contracts, and menu costs.

4. The probability of being visited by the Calvo fairy does not depend on the time since the last visit. There is thus a small chance that a firm is stuck with its 1917 price. This is mostly for mathematical convenience.

The price index, under Calvo pricing may be re-written as:

$$P_t = \left[\int_{s(t)} P_{t-1}(i)^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{14}$$

where $s(t)$ is the set of firms that were not able to re-optimize and P_t^* is the price chosen by those visited by the Calvo fairy. Because there are an infinite number of firms, the distribution of those unable to re-optimize will match the overall distribution. Thus:

$$P_t = \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{15}$$

Divide both sides of (15) by P_{t-1}:

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \tag{16}$$

Our solution methods depend on the system of equations being linear. Equation (16) is not. We thus take a linear approximation. We choose to take this approximation around the zero inflation steady state which is defined by $\Pi = 1$. Taking a first-order Taylor-Series expansion around this point yields:

$$(1-\epsilon)\Pi^{1-\epsilon} \frac{(Pi_t - \Pi)}{\Pi} = (1-\epsilon)(1-\theta)\Pi^{1-\epsilon} \frac{(\frac{P_t^*}{P_{t-1}} - \Pi)}{\Pi} \tag{17}$$

which simplifies to:

$$\frac{(\Pi_t - \Pi)}{\Pi} = (1-\theta) \frac{(\frac{P_t^*}{P_{t-1}} - \Pi)}{\Pi} \tag{18}$$

We now use the properties of logs to note that $\frac{X_t - X}{X} \approx ln(X_t) - ln(X)$. This and the fact that $ln(\Pi) = 0$ allows us to rewrite this as:

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{19}$$

where, as usual, lower case indicates logs.

The firm's optimal price setting problem can then be represented as follows:

$$max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k}(P_t^* Y_{t+k|t} - \Psi(Y_{t+k|t}))\right] \tag{20}$$

where θ^k is the probability that the firm will still be stuck with its re-optimized price in k periods. $Q_{t,t+k}$ is a stochastic discount factor. because the households own the firms,. we set it so that the marginal utility of profit in each period is equalized:

$$Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right) \tag{21}$$

where ψ is a generic cost function and $Y_{t+k|t}$ is the output of a firm in period $t+k$ that last re-optimized in period t.

s.t.

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} C_{t+k} \tag{22}$$

note that we may use Y_t and C_t interchangeably as there is no potential for out of equilibrium behavior.

We can re-state the problem by substituting (22) into (20):

$$max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k}((P_t^*)^{1-\epsilon} P_{t+k}^{-\epsilon} C_{t+k} - \Psi(Y_{t+k|t}))\right] \tag{23}$$

Differentiating with respect to P_t^* yields:

$$\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k}((1-\epsilon)(P_t^*)^{\epsilon} P_{t+k}^{-\epsilon} C_{t+k} - \Psi'(Y_{t+k|t})\frac{\partial Y_{t+k|t}}{P_t^*})\right] \tag{24}$$

As an instructor, I am now faced with a choice. We can continue through an elaborate and clever series of steps to restate (24) as a difference equation in only output and inflation. A lot of really useful mathematical tools are used in this process (linearizations, manipulation of summations, iterating, etc.). It is also exceptionally difficult and would take up about a week of class time. So while I am tempted, I am going to engage in some hand waving. But if you are highly motivated and have a wicked big brain, Gali takes you through it.

So, it can be shown that (24) may be re-stated as:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t \tag{25}$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta(1-\alpha+\alpha\epsilon)}(\sigma + \frac{\psi+\alpha}{1-\alpha})$.

Equation (25) is the *New Keynesian Phillips Curve*. It has the following properties:

1. It predicts that there exists a short-run tradeoff between more output and more inflation. Because prices are sticky, inflation will increase the demand for goods which are stuck with inefficiently low prices.

2. There is also a small long run tradeoff. This disappears as $\beta \to 1$. I have never really understood this but I am bothered that it, though small, exists. I will bump your grade in the class up by $\frac{1}{3}$ of a letter grade if you are able to explian it to me in a way that makes me really understand it.

3. $\tilde{y}_t$ is not output. It is the output gap, the difference between output and its flexible price level.

4. As $\theta \to 0$, prices approach full flexibility. $\kappa \to \infty$ implying that $\tilde{y}_t = 0$.

5. As $\theta \to 1$, κ gets smaller implying that fluctuations in inflation result in larger deviations of output from its flexible price level.

It is also possible to re-state the household's Euler Equation as:

$$\tilde{y}_t = E_t[\tilde{y}_{t+1}] - \sigma^{-1}(i_t - E_t[\pi_{t+1}] - r_t^n) \tag{26}$$

where r_t^n is the flexible price real interest rate.

Finally, it can be shown (basically from Ch.2) that the flexible price level of output depends only on TFP:

$$y_t^n = \frac{1+\psi}{\sigma(1-\alpha)+\psi+\alpha} a_t + \frac{(1-\alpha)\mu - ln(1-\alpha)}{\sigma(1-\alpha)+\psi+\alpha} \tag{27}$$

Equilibrium Dynamics

We close the model by assuming the following monetary policy rule:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \tag{28}$$

This is an example of *rules based monetary policy*. We can assume any rule we wish, (28) is surely not unique. We can for example, allow for expected or lagged inflation instead of current inflation. We could also include housing prices is we wanted to examine whether or not the fed should try to stabilize asset prices.[1] We assume that $\phi_\pi, \phi_y \geq 0$. The term v_t is a random shock to monetary policy.

In Chapter 2, we considered a similar rule without the output gap. Recall, however, that because monetary policy was neutral, there was no good reason to respond to output.

Inserting (28) into (27) yields a $2x2$ system of difference equations in π_t and $\tilde{y}_t$.

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} + \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \begin{bmatrix} 1 \\ \kappa \end{bmatrix} (r_t^n - \rho - v_t) \tag{29}$$

Equation (29) is a clever way of writing the model as a function of a single shock. It is not generally possible to do this for a model that includes two shocks.

We are only willing to consider solutions where both variables remain bounded. Note that neither variable is determined based on past variables (such as capital in a capital accumulation equation). We refer to such variables as *control variables*. To solve this model, we conduct an eigendecomposition. Write (29) as:

$$X_t = AE_t[X_{t+1}] + Ce_t \tag{30}$$

Now decompose A using $A = S\lambda S^{-1}$, premultiply by S^{-1} and define $Z_t = S^{-1}X_t$:

$$Z_{1,t} = \lambda_1 E_t[Z_{1,t+1} + d_1(r_t^n - \rho - v_t) \tag{31}$$

$$Z_{2,t} = \lambda_1 E_t[Z_{2,t+1} + d_2(r_t^n - \rho - v_t) \tag{32}$$

As usual, the eigendecomposition allows us to write the model as a set (in this case 2) independent difference equations). Recall that the condition for general stationarity of a forward looking difference equation is that λ_i has modulus greater than one. If this eigenvalue has modulus less than one, than $Z_{1,t}$ may behave explosively.

[1] Because the model does not include housing, however, housing prices are just a random variable, like v_t. It would therefore make no sense to target them. A serious analysis of this issue requires adding housing to the model.

If λ_i has modulus greater less than one, $Z_{1,t}$ will not behave explosively if agents choose it so that $E_t[Z_{1,t+1}] = 0$. In this case:

$$Z_{i,t} = d_i(r_t^n - \rho - v_t) \tag{33}$$

Equation (33) is a saddle condition. If there are two eigenvalues with modulus less than one, then there will be two such saddle conditions. In this case, there is only one equilibrium that describes the behavior of the two control variables. If, however, either or both eigenvalue has modulus greater than one, then indeterminacy of equilibrium occurs.

Taking the eigenvalues of A, both have modulus less than one if and only if:

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0 \tag{34}$$

This is the Taylor condition that ensures a unique equilibrium. If it is not satisfied, then sunspots may affect the economy undermining the monetary authority's stabilization goal.

If $\beta \approx 1$, then an approximate Taylor Condition is

$$\phi_\pi > 1 \tag{35}$$

which is the same as in Chapter 2. Once again, an aggressive response to inflation ensures a unique equilibrium.

We now consider the intuition behind indeterminacy. Suppose that the monetary authority screws up by setting $\phi_\pi < 1$. Now suppose that agents extraneously, due to a sunspot, come to expect higher inflation.

1. Note from (25) that inflation is highly persistent, increases in expected correlation are passed on to current inflation by nearly a one-to-one figure provides that β is close to one and the output gap is small enough.

2. Due to the policy choice, when $E_t[\pi_{t+1}]$ increases by one unit, π_t increases by about one unit, and i_t increases by less than one unit.

3. The real interest rate $i_t - E_t[\pi_{t+1}]$ thus decreases.

4. Based on the Euler Equation, households respond to lower interest rates buy increasing consumption.

5. Increased demand results in inflation. This makes the original extraneous expectation become self-fulfilling.

By satisfying the Taylor Condition, the monetary authority changes this series so that the real interest rate increases. The extraneous expectation is therefore no longer self-fulfilling.

We now examine how the economy responds to a random shock. To do this we calibrate the model which consists of plugging numerical values into the parameters. A good calibration will enjoy empirical and microeconomic support. Our calibration sets: $\beta = 0.99$, $\alpha = \frac{1}{3}$, $\theta = \frac{2}{3}$ (implying an average price duration of 9 months), $\sigma = 1$, $\epsilon = 6$, $\phi_\pi = 1.5$ (ensuring determinacy), $\phi_y = 0.125$, $\psi = 1$, $\eta = 4$, $\rho_v = 0.5$, and $\rho_u = 0.9$.

Monetary Policy Shock

To simulate the effects of a random change to the nominal interest rate, we assume:

$$v_t = \rho_v v_{t-1} + e_t^v \tag{36}$$

We consider a one-time increase in e_t.

Gali plots the results, known as impulse response functions. These include:

1. Both output and the output gap jump down and then converge back toward the steady state. Because prices are sticky, interest rates increase more than expected inflation. Real interest rates thus increase which reduces aggregate demand. The flexible price level of output is unchanged because monetary policy is neutral in this case.

2. Reduced demand puts downward pressure on prices which reduces inflation.

These impulse response functions can then be compared to those estimated from the actual data. As seen in Chapter 1, the general direction of the effects match those of the data. The effects in our model, however, occur too quickly. The model is said to lack persistence.

Marginal Cost Shock

To simulate the effects of a supply shock, we assume:

$$a_t = \rho_u a_{t-1} + e_t^a \tag{37}$$

A positive value of e_t^a reduces marginal costs. Unsurprisingly this reduces inflation and increases the flexible price level of output.

As seen in Gali, the output gap decreases in response to this shock. This may seem counterintuitive. This does not imply that output falls, instead it occurs because output increases by less than the flexible price level of output. This is because sticky prices prevent firms from lowering their prices by the efficient amount.