ECO 318: Theory on Finance and Growth[1]

I will rely on the following paper throughout these class notes:

Acemoglu, and F. Zilibotti. "Was Prometheus Unbound by Chance? Risk, Diversification, and Growth." *Journal of Political Economy*, vol. 105(4): 709-750.

Background

Within macroeconomics, the fields of growth and development are sometimes used almost interchangeably. There is a difference, however. Growth refers to the factors that affect an economy's long run macroeconomic performance. Development refers to how a country catches up to reach the technological frontier, for example, how do poor countries become rich countries. This paper has elements of both fields.

This paper addresses three fundamental macroeconomic questions:

First, why does growth suddenly take off so that a country escapes a long period of stagnation to then experience rapid growth? Here, the authors present a model where richer economies are better able to diversify risk. This incentivizes agents to pursue riskier investment opportunities that offer higher expected returns. As a result, growth rates are an increasing function of wealth.

The source of rich countries' better ability to diversify is the authors' assumption that investment projects are often indivisible. They need a certain amount of investment to have any chance of being successful. As a result, rich countries, by virtue of having more capital, will have access to more projects than poorer countries.

Second, why do poorer economies experience higher levels of volatility? Here, the inability to diversify means that they are exposed to greater risk than wealthy economies.

[1]These are undergraduate lecture notes. They do not represent academic work. Expect typos, sloppy formatting, and occasional (possibly stupefying) errors.

Third, is the time that it takes for an economy to become wealthy random or deterministic. Here, it is random. If an economy is lucky, in that its risky projects pay off, then it will grow faster, this leads to better diversification, and more growth. This is where the paper's title comes from.

The authors' argue that their model explains critical features of observed growth and development. For most of human history, there was little growth. Then a select group of countries (mostly in Europe and North America) were "lucky" in that they achieved a level of growth that allowed for them to take chances on riskier projects. For some countries, it worked out and rapid growth was then established. The authors use the example of American railroads in the nineteenth century as a risky project that paid off leading to sustained growth. Other countries, however, were unlucky in that their projects did not pay off. They then experienced disasters that delayed their development. The authors use the example of Spain where early investments in railroads fared poorly.

Data on the Relationship Between Wealth and Volatility

Good theory is often motivated by data. The authors present several results to illustrate the stylyzed fact that wealthier countries tend to have more stable growth rates. Some of the evidence that they provide is:

1. They cite existing work from economic history. McCloskey (1976) notes that in medieval England, famines occurred about every 13 years. This volatility would diminish as the country developed.

2. Figure 1 is a simple scatterplot showing GDP in 1960 along with the standard deviation of the growth rate from 1960-1985. A clear negative relationship exists.

3. Table 1 adds controls and performs a simple regression analysis. the same results comes through, all else equal, a poorer country is more volatile.

4. Quah (1993) calculates the probabilities that an economy in a given income range (he uses 5) will transition to a different range in the next period Transition rates fall as a country becomes wealthier and the wealthiest group rarely fall out of that group.

Having established this empirical relationship, the authors develop a theoretical, OLG growth model to help understand the possible causal factors at play.

Model

We begin with households. As usual, a continuum of identical households exist over the unit interval. As in our basic OLG model, households live for two periods, working in the first and living on their savings in the second. They are not altruistic.

Each household faces a standard choice of how much of their income to save and how much to consume. But the optimization problem has an extra layer. Each household must also decide how they save. They have two basic options:

1. They can save through a riskless asset that pays a guaranteed return r. Think of r as low.

2. Or they can choose among a continuum of risky assets defined $j \in [0, 1]$. If an agent saves F^j in risky asset j then this asset pays RF^j if and only if i) state j is realized with all states being equally likely, and ii) $F^j > M_j$. If either of these conditions are not satisfied the risky asset returns zero.

These risky assets are intermediate capital goods used in the production of a single final good that provides utility. The condition $F^j > M_j$ captures the notion in indivisibility. Some projects are only viable if their is enough investment in them. This assumption is critical to the entire paper. Intuitively, investing $1 total in a space probe will never yield any success. Obviously, a household will never choose a non-zero value of F^j that is less than M_j. Doing so would be throwing away wealth.

In period t, the household does not know what value of j will be realized in period $t+1$. This is how the authors add uncertainty into the model. If the realized value of j corresponds to a project that household invested in, then they benefit because $R > r$. If not, then, they would have , *ex-post*, been better off had they invested only in the safe asset.

Suppose that all outcomes (realized values of j) are equally likely. Denote p as the fraction of projects households have invested in (because all households are the same, they all make the same choices). Then the following properties hold:

1. If $p = 0$, then r is also the aggregate return on savings.

2. With probability p, households return R on their risky assets. With probability $1-p$, they return 0.

3. If $p = 1$ then households return R with certainty. Households would like to choose this option. They can do so, however, only if they have enough capital to cover the indivisibility requirement for all projects. In the model, only sufficiently advanced economies will be able to do so.

The authors then impose structure on M_j:

$$M_j = max\{0, \frac{D}{1-\gamma}(j-\gamma)\} \tag{1}$$

In words, for $j \leq \gamma$, there is no indivisibility requirement. Households will always invest in these projects. For $j > \gamma$, the indivisibility requirement matters and the parameter D determines how important it is.

Households' must decide how many intermediate goods to invest in. Once they have made this choice, they will minimize their risk exposure by distributing their savings equally across all such projects. They do so because the projects are otherwise identical.

Although solving the model is complicated (and we will largely skip the technical details), the rest of the model is fairly standard. Households now maximize their expected (because there is uncertainty) utility:

$$E_tU(c_t, c_{t+1}) = log(c_t) + \beta \int_0^1 log(c_{t+1}^j dj \tag{2}$$

Here, the integral just reflects that households are uncertain about which state, j, will be realized.

Production of the final good is Cobb-Douglas: $Y_t = AK_t^{\alpha}L_t^{1-\alpha}$, where capital equals the following if households invest in the realized state:

$$K_{t+1} = \int (r\phi_t + RF_t^j) \tag{3}$$

and ϕ_t is the amount invested in the safe asset. Capital in $t{+}t$ equals the following if households do not invest in the realized state:

$$K_{t+1} = \int (r\phi_t) \tag{4}$$

Capital and labor markets are competitive so that inputs are paid their expected marginal products. The authors begin by showing the static equilibrium (for a period with a pre-determined capital stock). Figure 3 shows the main result and is quite intuitive.

The first key function is denoted $aF^*(n_t)$. This is the amount that households would like to invest in each asset, ignoring the indivisibility requirement. Critically, this is an increasing function of n_t, the number of sectors that households are investing in. Intuitively, as more sectors are being used, households are better able to diversify. They thus move away from the safe asset and invest more in each risky asset. The second key function is M_n. Where the two meet, equilibrium occurs.

Implications for Finance

We can think of the finance sector has having two potential impacts. A better financial sector might imply that D is lower or γ is higher. Both will reduce the issue posed by indivisibility, increasing the amount of investment in risky projects. Because risky projects have a higher expected return than riskless projects, this will increase the expected growth rate.

We should be careful not to make any welfare implications from the previous paragraph. Agents would be better of if D was lower or γ is higher. But this is a somewhat crude way of thinking about the finance sector. In reality, finance takes resources which could be used elsewhere. Without explicitly modelling this process, we shouldn't say much about welfare.

Dynamic Equilibrium

The authors now turn to the dynamic equilibrium, characterized by the capital accumulation process. The key is that it is generally. Lucky draws cause the capital stock to display additional growth while unlucky draws cause it to drop.

Because it is random, the model does not have a steady state like in the ordinary, deterministic OLG model. The authors define a "quasi bad steady state." If an economy is always unlucky, it will converge to this steady state. Because of the Inada conditions, and because households choose to save a positive amount in the riskless asset, this is a positive level of the capital stock. The authors define another steady state, K^{SS} in the opposite manner, the point where an economy converges if it always lucky. To get here, the economy passes the point where $n = 1$. All uncertainty is removed. K^{SS} is thus a true steady state, if the economy gets here, it will never leave.

Proposition 2: As $t \rightarrow \infty$, the economy will surely converge to K^{SS}.

Proof (by intimidation): It is obvious even to the most simple-minded reader. If you don't see this, hang your head in shame.

Intuition: Eventually, every economy will have a string of good luck that takes them pass the point where $n = 1$. Convergence to K^{SS} will then occur.

The authors next do a calibration exercise where they determine how long it takes for the economy to converge, defined as when all sectors are open and the economy is thus fully diversified. For the case where $\alpha = 0.35$, convergence takes 19.5 periods on average. But the 90% confidence interval ranges between 7 and 30 periods. For larger values of α, convergence is slower.

This takes us back to the paper's title. The development process is random. Two identical economies can display dramatically different processes. And an economy that seems to be on the path to full development, can have this process disrupted by a period of bad luck. the authors discuss the case of England in the 1850s by noting that the technology for the industrial revolution to begin was already available centuries earlier. According to the model, its occurrence in the mid-nineteenth century was random. It could just as easily have happened earlier.

Volatility

Proposition 3 shows that there are two possible relationships in the model between growth and volatility:

i. If R is big enough, or if γ is large enough (both of which incentivize households towards risky projects), then as the economy grows, the variance of the growth rate uniformly decreases. In this case, the model globally fits the data.

ii. Otherwise, the relationship takes the shape of a parabola where the variance of growth is initially increasing before beginning to decrease for higher levels of development.

Other Results

1. The model contains a positive externality. Each household's investment in risky activities helps open more sectors which allows other households to better diversify. Because households do not take this into account, investment into risky assets is inefficiently low. It then follows that (average) growth is inefficiently low and it takes longer (on average) for convergence to K^{SS}.

2. The authors consider whether more complicated financial instruments, such as mutual funds where agents buy shares of a financial intermediary which then invests in projects, could overcome this externality. Surprisingly, they find that such instruments can not.

3. Finally, the authors extend the model to a two-country setting. Here, they find that for low levels of development, capital flows from poor to rich, contributing to divergence. However, for higher levels of development, this result is reversed causing convergence. They argue that this pattern fits the historical evidence.