Midterm Key 2017

1 a.) No. The process is mean-stationary. But it is neither variance nor co-variance stationary.

Iterating backwards

$$\varepsilon_t = e_t + \frac{1}{\alpha_{t+1}} e_{t-1} + \frac{1}{(\alpha_{t+1})(\alpha(t-1)+1)} e_{t-2} \ldots$$

$$\text{Var}(\varepsilon_t) = \sigma_\varepsilon^2 + \left(\frac{1}{\alpha_{t+1}}\right)^2 \sigma_\varepsilon^2 + \left(\frac{1}{(\alpha_{t+1})(\alpha(t-1)+1)}\right)^2 \sigma_\varepsilon^2$$

which clearly depends on t.

b.) $X_t = \frac{1}{2} X_{t-1} + t^2 + e_t$

Differencing

$$\Delta X_t = \frac{1}{2} X_{t-1} + t^2 + e_t - \frac{1}{2} X_{t-2} - (t-1)^2 - e_{t-1}$$

$$\Delta X_t = \frac{1}{2} \Delta X_{t-1} + e_t - e_{t-1} + 2t - 1$$

which is still non-stationary.

C.) Lags work as a "quasi-instrument"

X_{t-1} is usually correlated with X_t

It is less likely to be correlated with the error term than X_t, though it may still be.

d.) R-S episodes are seemingly large and exogenous shocks that lead to changes in G and T. It is a natural way to deal with endogeneity.

2 a.) Note: there is more than 1 way to model this.

Define $\Delta i_t = \Delta (i_t^h - i_t^f)$, the interest rate spread

$\Delta S_t = \Delta (\ln(f_{t,t+1}) - \ln(e_t))$, the change in the futures market's forecasted appreciation

Both are differenced to deal with non-stationarity

$$y_t = \alpha + B y_{t-1} + u_t$$

where $$y_t = \begin{bmatrix} \Delta i_t \\ \Delta S_t \end{bmatrix}$$

b.) Brute force. Try different lags and then use an information criteria to select among them.

c.) True. Although a seemingly unrelated regression (SUR) model is generally more efficient when there are common independent variables, when the RHS is exactly the same,

OLS = SUR.

d.) $\Sigma = \begin{bmatrix} 1 & -.5 \\ -.5 & .1 \end{bmatrix}$

$y_t = \alpha + B\, y_{t-1} + \Theta^{-1} \varepsilon_t$

$\Theta^{-1} = \begin{bmatrix} c & 0 \\ d & e \end{bmatrix}$

$$\begin{bmatrix} 1 & -.5 \\ -.5 & 1 \end{bmatrix} = \begin{bmatrix} c & 0 \\ d & e \end{bmatrix} \begin{bmatrix} c & d \\ 0 & e \end{bmatrix}$$

$c^2 = 1 \qquad cd = -.5 \qquad d^2 + e^2 = .1$

$c = 1 \qquad d = -.5 \qquad e = \sqrt{-.15}$ ← WTF?

A shock to $e_{1,t}$

$y_t = \begin{bmatrix} 1 \\ -.5 \end{bmatrix}$

A shock to $e_{2,t}$

$y_t = \begin{bmatrix} 0 \\ \sqrt{-.15} \end{bmatrix}$

↑ imagine it.

e.

$$\theta^{-1} = \begin{bmatrix} c & d \\ d & e \end{bmatrix}$$

$$\begin{bmatrix} c & d \\ d & e \end{bmatrix}\begin{bmatrix} c & d \\ d & e \end{bmatrix} = \begin{bmatrix} 1 & -.5 \\ -.5 & .1 \end{bmatrix}$$

$$c^2 + d^2 = 1 \qquad cd + e^2 = -.5$$

$$d^2 + e^2 = .1$$

→ IRFs are then calculated in the same manner as d)

f.) (1) shows there is clear - cointegration

co integrating vector is $i_t + \gamma s_t$

Run a vector error correction model

$$y_t = \begin{bmatrix} i_t \\ s_t \end{bmatrix}, \quad y_t = \alpha + B y_{t-1} + \delta(i_t + \gamma s_t) + u_t$$

3. a.) This is pretty open ended. The issue is whether there is unobserved heterogenity and if it is correlated with the independent variables. I suspect that some cities have chronically higher crime rates than others and it is not explained only by guns and population. This pre-cludes pooled OLS.

I also suspect that it is correlated with guns and population. I thus suspect fixed effects is correct.

Any good discussion got full credit no matter which model was proposed.

b.) An f-test, jointly conducted on all fixed effects is used to rule out pooled OLS.

A - Hausman test is then used to test between fixed and random effects.

Because $N=3$, year fixed effects are not feasible. For time fixed effects, I might instead try group (decade ?) fixed effects.

c-e.) My ordering

1) Clear non-stationarity by including population. I could difference. But I would instead divide murders and guns by population to make them per-capita. Missing observations probably rule out formal unit root tests

2.) Endogeneity. Do guns cause crime or does crime cause guns. I could seek an instrument for guns. Or I could use the lag of guns.

3.) Year fixed effects eat up ≈ 30 degrees of freedom and are probably inefficient.

4.) The effects of guns may be with a long lag. I could try adding more lags or consider a PVAR.

- I didn't list omitted variable bias because X_{it} is an "appropriate" set of controls. I would ample partial credit, however, if you made a reasonable argument for what should be in X_{it}.