Linear Algebra and Econometric Review: Key

Consider the following matrices:

$$A = \begin{bmatrix} 3 & 9 \\ 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 \\ 1 & 4 \\ 5 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 3 \end{bmatrix}$$

1. Calculate the following:

a.

$$\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 1 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 43 & 23 \\ 16 & 14 \end{bmatrix} \tag{1}$$

b.

$$\begin{bmatrix} 4 & 1 \\ 1 & 4 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 1 & 31 \\ 2 & 4 & 19 \\ 10 & 3 & 44 \end{bmatrix} \tag{2}$$

c.

$$\begin{bmatrix} 3 & 9 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 43 & 23 \\ 16 & 14 \end{bmatrix} = \begin{bmatrix} 46 & 32 \\ 17 & 17 \end{bmatrix} \tag{3}$$

d. This sum does not exist. The matrices are of different dimensions

e.

$$\begin{bmatrix} 3 & 9 \\ 1 & 4 \end{bmatrix}^{-1} = \frac{1}{3} \begin{bmatrix} 4 & -9 \\ -1 & 3 \end{bmatrix} \tag{4}$$

f.

$$Det(A) = 12 - 9 = 3 \tag{5}$$

g. Because C is not a square matrix, its inverse is not defined.

h. the eigenvalues and eigenvectors of CB

$$Det \begin{bmatrix} 43-\lambda & 23 \\ 16 & 14-\lambda \end{bmatrix} = 0 \tag{6}$$

$$(43-\lambda)(14-\lambda) - 23 * 16 = 0 \tag{7}$$

$$\lambda^2 - 56\lambda + 191 = 0 \tag{8}$$

$$\lambda = \frac{56 \pm \sqrt{56^2 - 4 * 191}}{2} = 3.65, 52.35 \tag{9}$$

for $\lambda = 3.65$:

$$\begin{bmatrix} 43-3.65 & 23 \\ 16 & 14-3.65 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} 39.35x_1 + 23x_2 \\ 16x_1 + 9.35x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.58 \end{bmatrix} \tag{12}$$

for $\lambda = 52.35$:

$$\begin{bmatrix} 43-52.35 & 23 \\ 16 & 14-52.35 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} -9.35x_1 + 23x_2 \\ 16x_1 + -39.35x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2,46 \end{bmatrix} \tag{15}$$

i. Because C is not square, the eigenspace is not defined.

j. Because B is not a square matrix, its inverse is not defined.

2. Consider the following model:

$$y_t + \alpha\pi_t = e_t \tag{16}$$

$$\pi_t = u_t \tag{17}$$

where y_t is output and π_t is inflation, both endogenous variables, and e_t and u_t are exogenous shocks so that:

$$Dx_t = \mu_t \tag{18}$$

a.

$$\begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} e_t \\ u_t \end{bmatrix} \tag{19}$$

b.

$$D^{-1} = \begin{bmatrix} 1 & -\alpha \\ 1 & 0 \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & -\alpha \\ 1 & 0 \end{bmatrix} \begin{bmatrix} e_t \\ u_t \end{bmatrix} \tag{21}$$

3. Note that $\frac{dollar/yen}{dollar/euro} = \frac{1}{yen/euro}$. The specification thus exhibits perfect multicolinearity and can not be estimated. The solution is to drop one of the three exchange rates. This does not cause omitted variable bias because the two remaining exchange rates have just as much explanatory power as all three exchange rates.

4. This is a rather open ended question. The first part is yes. Your answer should at least mention omitted variable bias which can never fully be eliminated. The answer to the second part is surely no.

5. All regressions omit some variables, these omitted variables are included in the error term. Suppose, for example,temperature is omitted. Because one hot day is often followed by another, this variable will be positively autocorrelated. This autocorrelation is thus transferred to the error terms.

6. This specification probably suffers from endogeneity because profits and share price may be simultaneously determined. Suppose, for example, that there is a macroeconomic shock that benefits the firm. This will likely cause both a positive error term and a higher level of profits. The independent variable is thus correlated with the error term. The solution is typically to find an instrument that is correlated with the independent variable but is uncorrelated with the error term.

7. False. R^2 is a measure of fit, how much of the variation in the dependent variable is explained by variation in the independent variables? An information criterion weights a measure of fit against the number of right hand side variables.

8. I am torn on this one.