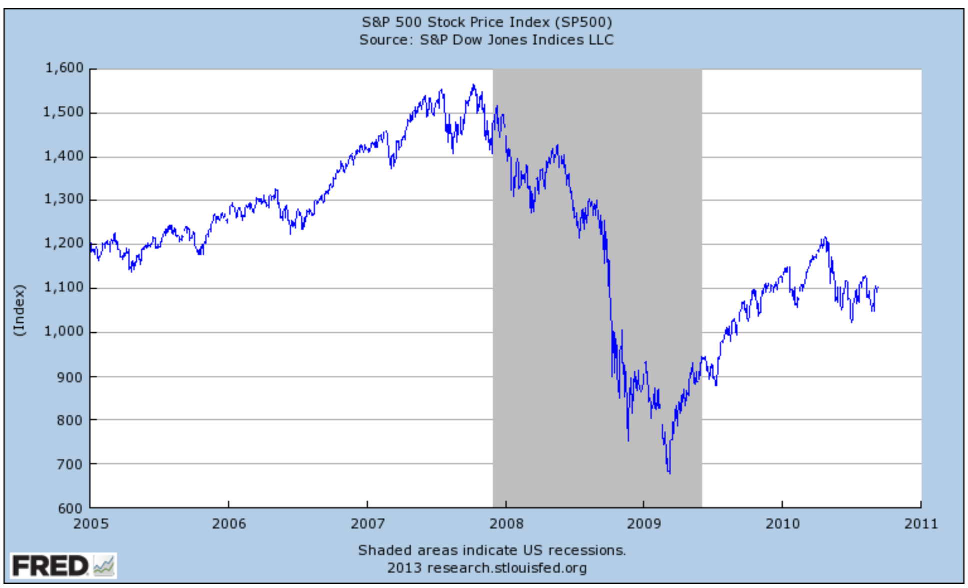
Introduction: Problems

1. Re-write the following stochastic process using lag operators:

$$x_t = \delta + (\alpha_1 L + \alpha_3 L^3 + \alpha_4 L^4)x_t + (1 + \theta_1 L + \theta_3 L^3)u_t \tag{1}$$

2. OLS may be appropriate if the temporal component of the data does not matter. For example, suppose that I run the exact same physics experiment every year for ten years and this yields my time series. Because the laws of physics are time invariant there is no relationship among experiments. As long as there are no other issues with OLS, we can safely ignore that the data are time series.

Consider the following graph of the S&P 500 from 2005-2011:



3. Ergodicity refers to whether or not the sample distribution converges to the true distribution as the sample size approaches infinity. These are sample data. We cannot observe the true distribution. Ergodicity is thus something that must be assumed and we cannot test for it, either by eyeballing the data or by running more sophisticated econometrics.

4. To me, based only on these data and not any additional understanding of stock prices, it seems possible that this process is mean stationary. It also seems clear, however, that the later period (after the financial crisis) is considerably more volatile than the earlier period. The

variance thus seems variance non-stationary and the time series thus probably does not qualify even as weakly stationary.

Consider the following time series:

$$x_t = \delta + tu_t \tag{2}$$

where u_t is white noise with mean equal to $\bar{u}$

5. Yes. For any period, the true mean of the time series is $\bar{u}$.

6. No. For any period, the true variance of the time series is $t^2\sigma_u^2$. This is clearly time dependent.

7. False. Stationarity refers to the true distribution, not the sample distribution. In this case, the different sample distributions results from the inherent randomness of the time series, not from time dependence.

8. Very true.