Growth: Production and the Solow Model: Key

1. Obviously this is a subjective question. In my opinion, the assumption that total factor productivity is exogenous is weak. By making this assumption, we limit the model's ability to discuss events or polices which affect elements of TFP such as technological innovation or human capital. The next model of the class, Endogenous Growth, endogenizes TFP.

2. If we are working with total (not per-capita) variables then the result is ambiguous. If, for example, the change in TFP is large while the change in L is small, then the production function will likely shift upwards, suggesting that any amount of labor is more productive than before. But if the opposite is true, then the production function will likely shift downwards. I have found that undergraduates hate questions where ambiguity is the correct answer. But a good model may produce such a result.

If we are working with per-capita variables, then the change to L has no effect on the production function. It therefore shifts up as TFP increases.

3. In my opinion, this is false. The Solow Model's objective is to explain long run macroeconomic performance. Simplicity is a worthy goal of any model. The Solow Model assumes away cyclical unemployment in order to address its central question with more clarity. Were it a model of business cycles, then this approach would be dubious.

4a. A decrease in TFP has the following effects:

i. Both the production function and savings function shift down.

ii. The steady state levels of y, k, and c are each reduced. This is easily demonstrated using either a graph or algebra (recall we signed these derivatives).

iii. The economy will thus initially be above its steady state level of capital. As it converges toward the new steady state, all three endogenous variables will decline.

4b. A decrease in d has the following effects:

i. The depreciation function becomes flatter.

ii. The steady state levels of y, k, and c are each increased. This is easily demonstrated using either a graph or algebra (recall we signed these derivatives).

iii. The economy will thus initially be below its steady state level of capital. As it converges toward the new steady state, all three endogenous variables will increase.

Suppose that the production function takes the following functional form:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

5. Dividing both sides of (1) by L yields:

$$\frac{Y_t}{L_t} = y_t = A_t K_t^{\alpha} L_t^{1-\alpha} L_t^{-1} = A_t K_t^{\alpha} L_t^{-\alpha} = A_t k_t^{\alpha} \tag{2}$$

6. Recall the capital accumulation equation:

$$K_{t+1} = (1 - \bar{d})K_t + sY_t \tag{3}$$

Insert (1) into (2)

$$K_{t+1} = (1 - \bar{d})K_t + sA_t K_t^{\alpha} L_t^{1-\alpha} \tag{4}$$

To reflect the steady state, substitute K into (4) and drop time subscripts on the other variables:

$$K = (1 - \bar{d})K + sAK^{\alpha} L^{1-\alpha} \tag{5}$$

Subtract $(1 - \bar{d})$ from both sides, then divide both sides by $\bar{d}$:

$$K = \frac{sAK^{\alpha} L^{1-\alpha}}{\bar{d}} \tag{6}$$

Multiply both sides by $K^{-\alpha}$:

$$K^{(1-\alpha)} = \frac{sAL^{1-\alpha}}{\bar{d}} \tag{7}$$

Raise both sides to the power $\frac{1}{1-\alpha}$:

$$K = (\frac{sA}{\bar{d}})^{\frac{1}{1-\alpha}} L \tag{8}$$

Now insert (8) into (1):

$$Y = AL^{1-\alpha} (\frac{sA}{\bar{d}})^{\frac{\alpha}{1-\alpha}} L^{\alpha} \tag{9}$$

which is hideously messy. Cleaning up exponents yields:

$$Y = A^{\frac{1}{1-\alpha}} (\frac{s}{\bar{d}})^{\frac{\alpha}{1-\alpha}} L \tag{10}$$

To obtain per capita output, divide by L:

$$y = A^{\frac{1}{1-\alpha}} (\frac{s}{\bar{d}})^{\frac{\alpha}{1-\alpha}} \tag{11}$$

To obtain per capita consumption, multiply by $(1 - s)$, the fraction of output not saved:

$$c = (1 - s) A^{\frac{1}{1-\alpha}} (\frac{s}{\bar{d}})^{\frac{\alpha}{1-\alpha}} \tag{12}$$

Per capita capital equals:

$$k = (\frac{sA}{d})^{\frac{1}{1-\alpha}} \tag{13}$$

7. Taking a first-order Taylor Series expansion:

$$\tilde{k}_{t+1} = (1 - d)\tilde{k}_t + \alpha s A k_t^{\alpha-1} \tilde{k}_t \tag{14}$$

Inserting (13) into (14):

$$\tilde{k}_{t+1} = (1 - d(1 - \alpha))\tilde{k}_t \tag{15}$$

As before, higher values of d result in quicker convergence. Higher values of α slow convergence.

8. This version is more general and allows us to do more experiments. We can, for example. analyze how a decrease to labor's share of income affects the steady state. It is also more complicated.

9. Uncertain. As discussed in class, the optimal (golden rate) of saving is between 0 and 1. It is possible to save too much.

10. Money is not included in the model. So even the wise guidance of Gary would have no effect.