ECO 270: Aggregate Production and the Solow Model[1]

This represents our first attempt at a formal theoretical model. Most students will find this topic much more challenging than the material we have covered to date. Before beginning, recall that models are judged both by the quality of their assumptions and the accuracy of their empirical predictions. Having reviewed the Barro paper, we are in a position to evaluate this model in both manners.

Chapter 4 of Jones develops a model of the *aggregate production function*. This model will be part of the Solow Model, which Jones covers in Chapter 5. I am thus treating both chapters as a single topic. The aggregate production function describes how a collection of aggregate inputs results in a level of aggregate output.

We begin by listing a set of assumptions about the aggregate production function:

1. There are three inputs. The first is labor supply (L). The second is physical capital (K). The third is total factor productivity (A).[2] Total factor productivity (hereafter TFP) refers to all inputs that affect output (Y) except labor and capital. TFP and technology are sometimes used interchangeably. But more generally, TFP may include other inputs such as human capital, energy, and land.

2. An important assumption in the Solow Model is that TFP is exogenous. The model is thus making no effort to explain why TFP equals a particular value. This is reasonable if we believe that neither policy nor the choices of agents in the model affect its value. Because TFP is exogenous, the Solow Model is a *classical* growth model. Some economists vehemently disagree with this assumption. One group makes TFP endogenous in what has come to be known as the Endogenous Growth Model. We will study this model after the Solow Model.

3. Labor is fully employed and capital is fully utilized. The former assumption eliminates unemployment. At first glance, this may seem odd for a macroeconomic model. But our goal is to explain long run GDP, not short run fluctuations, that is left for models of the business cycle. It is therefore accurate to think of this as a model of potential GDP (GDP if unemployment equals its natural rate), rather than actual GDP. If capital is not fully utilized, then changes in its utilization rate become part of TFP.

Combining #1-3, we may represent the production function as:

[1]These are undergraduate lecture notes. They do not represent academic work. Expect typos, sloppy formatting, and occasional (possibly stupefying) errors.

[2]The text includes bars over several variables to indicate that they are exogenous. I have dropped these bars in my notes.

$$Y = AF(K, L) \tag{1}$$

TFP is also known as the Solow Residual. Suppose that we have data on Y, K, and L, along with a functional form for $F(K, L)$. We can then calculate TFP as the exact value of A that allows (1) to hold with equality. In other words, TFP is just the unexplained part of the production function.

In the Great Recession, for example, both K and L declined, but not by enough to explain the observed decrease in Y for a fixed value of A. By definition, TFP therefore declined so that (1) hold with equality. We should thus not necessarily interpret decreasing TFP as a cause of the economic downturn.

4. $F(K, L)$ is increasing in both K and L. More inputs yield more output. Formally:

$$\frac{\partial F}{\partial K} > 0, \quad \frac{\partial F}{\partial L} > 0 \tag{2}$$

5. Both K, and L exhibit diminishing returns. Formally:

$$\frac{\partial^2 F}{\partial K^2} < 0, \quad \frac{\partial^2 F}{\partial L^2} < 0 \tag{3}$$

Suppose that there is initially one unit of both capital and labor. Further suppose that this results in one unit of output. Now suppose that we add a second unit of labor. Initially, each unit of labor had one unit of capital to work with. Now each unit of labor has only $\frac{1}{2}$ units of capital. We are assuming that this will make each unit of labor less productive so that they each produce less than one unit of labor apiece. Aggregate output should therefore increase above 1 (See #4), but it should not double. An analogous argument applies to capital.

6. The production function exhibits *constant returns to scale*. If we double both K and L, we double Y. Formally, for any K and L:

$$F(\alpha K, \alpha L) = \alpha F(K, L) \tag{4}$$

The defense for #6 is the *standard replication argument*. Suppose that one factory produces a certain amount of output. Now suppose that I add an identical (in terms of capital and labor) factory to the economy. Should output increase by the amount of the initial factory's production?

There are good reasons why this argument does not hold. Perhaps the two factories could learn from each other and thus increase each others' productivity. For now, however, we will work with this assumption.

7. If $K = 0$ or $L = 0$, then $F(K, L) = 0$. This is mostly for mathematical convenience and to make our graphs look pretty.

These assumptions cause the aggregate production function to take a specific shape when graphed. In growth models, it is common to plot output on the vertical axis and capital on the horizontal.[3]

Graph: The Aggregate Production Function:

#7 causes the production function to pass through the origin. #4 causes it to be upward sloping. #5 causes it to become flatter as K increases (the production function is said to be concave). The previous graph is for given values of K and A. If labor increases, then any amount of capital yields more output. The production function then shifts upwards. Likewise if TFP increases, perhaps through technological advancement, then it also shifts upwards. If either L or A decrease, then the production function shifts downward.

We now further assume that the production function takes the following specific form:

$$Y = AK^{\frac{1}{3}}L^{\frac{2}{3}} \tag{5}$$

You should verify that (5) is consistent with #1-7. Note, however, that it is just one of an infinite number of production functions to be consistent with our assumptions. Before proceeding, we thus explore its empirical fit.

Consider a perfectly competitive economy. Firms take the prices of labor (the wage, w) and capital (the rental rate, r) as given because they are too small to affect these variables. A profit maximizing

[3] When we graph business cycle models, however, labor will typically be on the horizontal axis instead of capital. This is because growth models tend to explain deviations in output using changes to K while business cycle models tend to do so based on changes to L.

firm thus solves the following problem:[4]

$$Max_{K,L} \ \ AK^{\frac{1}{3}}L^{\frac{2}{3}} - rK - wL \tag{6}$$

Differentiating with respect to K and L, and setting these terms equal to zero, yields the demands for capital and labor:[5]

$$MPK = A\frac{\partial F}{\partial K} = \frac{1}{3}A(\frac{L}{K})^{\frac{2}{3}} = \frac{1}{3}\frac{Y}{K} = r \tag{7}$$

$$MPL = A\frac{\partial F}{\partial L} = \frac{2}{3}A(\frac{K}{L})^{\frac{1}{3}} = \frac{2}{3}\frac{Y}{L} = w \tag{8}$$

Now suppose that we consider total labor income in the economy. This is simply wL. Using (8), this equals $\frac{2}{3}Y$. Likewise, income paid to the owners of capital is $rK = \frac{1}{3}Y$. These are known as factor shares. Fortunately, data on these are easily available. They show that these factor shares have been surprisingly stable throughout most of U.S. history.[6] And, as predicted by our model, labor's share has been about $\frac{2}{3}$. This supports our assumption from (5). It also explains why we chose $F = K^{\frac{1}{3}}L^{\frac{2}{3}}$ instead of $F = K^{\frac{1}{2}}L^{\frac{1}{2}}$. the latter predicts that each inputs' factor share equals one-half.

The Intensive Production Function

Equation (1) describes real GDP. When comparing living standards, we are more interested in per capita GDP. Assuming a constant labor force participation rate, plausible in the long run, we convert real GDP to real per capita GDP by dividing both sides by L.[7] Define $y = \frac{Y}{L}$ as real per capita GDP.

$$y = \frac{Y}{L} = AL^{\frac{-1}{3}}K^{\frac{1}{3}} = Ak^{\frac{1}{3}} \tag{9}$$

where $k = \frac{K}{L}$, per-capita capital. Table 4.3 in Jones Ch. 4 performs an interesting exercise. Suppose that we collect data on per-capita capital and output. For the U.S., it normalizes this so that one unit of capital yields one unit of output. Now collect the same data for Burundi. Capital per person in

[4]In solving this problem, I am assuming that there are a large number of identical firms, and I am solving for one such firm. This is known as a representative agent.

[5]We are being lazy and not checking second order conditions. They do hold.

[6]Recently, however, labor's share has started to decline. If this persists, then we may have to revisit the validity of the Cobb-Douglas functional form.

[7]During the recent recession, unemployment and reduced labor force participation create a bigger gap between output per worker and per-capita output. Keep in mind, however, that the Solow Model is only trying to explain the long run (potential output, not actual output).

Burundi is 0.3% of that in the United States. Inserting this figure into (9) predicts that per capita output in Burundi is 14.9% of that in the United States.

$$\frac{y^{US}}{y^{BUR}} = (\frac{k^{US}}{k^{BUR}})^{\frac{1}{3}} = (\frac{1}{0.003})^{\frac{1}{3}} = \frac{1}{0.149} \tag{10}$$

In actuality, however, it is only 1.5% as large. Doing the same exercise for other countries yields additional erroneous predictions. We can thus conclude that differences in capital per person alone cannot explain the actual cross country distribution of income. There must be more to the story.

Figure 4.4 performs another exercise. Normalizing $A_{U.S.} = 1$, it calculates the level of TFP in other countries needed to perfectly explain the observed levels of per capita income. For Burundi, we simply divide the predicted level of per capital output (0.149) with the actual (0.015) to obtain $A_{Bur} = .101$. In other words, inputs in Burundi are about 10% as productive as in the United States.

It is not obvious whether such TFP differentials are plausible when we assume that TFP is exogenous. If they are not, then are assumption that A is exogenous seems highly dubious.[8] In Chapter 4.4, Jones presents some factors that might plausibly explain such differentials:

1. Technology. Clearly the United States is more advanced than Burundi. Note that technology refers not just to the most sophisticated production process that exists in the economy, but also how widespread state of art technology is in each economy.

2. Institutions. This is closely related to the variable of rule of law from Barro's paper. Despite any overheated rhetoric that you may hear, the U.S. has long had relatively effective courts and other institutions that protect property rights, as well as fairly low levels of corruption. Burundi, however, has long been plagued by corruption, ineffective government, and war.

3. Human capital. This concept is closely related to education. The United States has a much higher level of educational attainment than Burundi.

The Solow Growth Model

It is a stretch to call the model of the production function a true model of economic growth. Although it does predict a level of output, it does so only for given values of TFP, labor, and capital. These values are all exogenous. A more interesting approach is to endogenize one of these inputs.

[8] I really wanted to use "bogus" here in honor of *Bill and Ted's Bogus Journey*. I then realized that some of you were not even born when that movie hit theaters.

The Solow Model does this by endogenizing capital. Capital is a result of the Solow Model, its value is not simply taken as given.

The Solow Model is an old model. Nobel Prize winning macroeconomist Robert Solow of MIT published it in the 1950s. It remains a useful starting point for the formal study of growth models. Versions of the Solow Model may employ either the aggregate production function or the intensive production. We will rely on the latter.

The Solow Model makes capital endogenous. In doing so, output and consumption also become endogenous. Our goal is to solve for these endogenous variables as a function of exogenous variables and parameters such as TFP. Our tools will be calculus, algebra, and graphs.

The Solow Model adds two assumptions to our production model. First, a constant fraction of output is saved. Because Y is total (real) output, sY is total savings where s represents the savings rate.

Recall the national income identity from Econ 103:

$$Y \equiv C + I + G + NX \tag{11}$$

To keep our model as simple as possible, we set $G = T = NX = 0$. Without taxes, all household income is either saved or consumed: $Y = S + C$. Total savings is thus $Y - C$. Using (11) it follows that:

$$I = sY \tag{12}$$

Investment is thus equal to total savings because there is go government or trade. Recall that investment refers to the creation of new capital.[9] The second assumption that the Solow model makes is to assume that a constant fraction, d, of capital depreciates each period. The *capital accumulation equation* may then be written as:

$$K_{t+1} = (1-d)K_t + sAK_t^{\frac{1}{3}}L_t^{\frac{2}{3}} \tag{13}$$

The first term on the right hand side of (13) is undepreciated capital from the previous period. The second term is investment (note that it equals sY_t). We assume that investment takes one period to become new capital.

[9] Consider a person buying stock. In common English, this is often called investment. In macroeconomics, however, this is saving.

We can solve for either total variables (*e.g.* Y_t), or per-capita variables (*e.g.* $y_t = \frac{Y_t}{L_t}$). Because our main goal is to compare living standards, we choose the latter.

For simplicity, assume that the labor force is constant so that $L_t = L$ for all t. Dividing both sides of (13) by L then yields:

$$k_{t+1} = (1-d)k_t + sAk_t^{\frac{1}{3}} \tag{14}$$

Equation (14) is the intensive form of the capital accumulation equation.

Our first step in solving the model is to analyze the model's *steady state*. A steady state is a long run equilibrium where, if the model's variables equal a certain set of values, then they will never again change from these values. In other words, $k_{t+1} = k_t = k$, $y_{t+1} = y_t = y$, etc.[10] To find the steady state, we substitute $k_{t+1} = k_t = k$ into (14):

$$k = (1-d)k + sAk^{\frac{1}{3}} = (\frac{sA}{d})^{\frac{3}{2}} \tag{15}$$

To find the steady state level of output, we note that $y = Ak^{\frac{1}{3}}$. Inserting (15) into this term yields:

$$y = A^{\frac{3}{2}}(\frac{s}{d})^{\frac{1}{2}} \tag{16}$$

Finally, we solve for steady state consumption by noting that a constant share, s of (16) is saved. It follows that a constant share, $(1-s)$, is consumed.

$$c = (1-s)A^{\frac{3}{2}}(\frac{s}{d})^{\frac{1}{2}} \tag{17}$$

We can now examine how the steady state values of these three variables depend on the parameters A, d, and s. This entails differentiating (15)-(17) with respect to the parameter of interest, and checking the sign. The following table does so. Although it may look scary, it derivation requires only simple calculus:

Table 1: Comparative Statics in the Solow Model

	$\partial y/$	$\partial k/$	$\partial c/$
∂A	$\frac{3}{2}(\frac{SA}{d})^{\frac{1}{2}} > 0$	$\frac{3}{2}A^{\frac{1}{2}}(\frac{s}{d})^{\frac{3}{2}} > 0$	$(1-s)\frac{3}{2}(\frac{sA}{d})^{\frac{1}{2}}$
∂d	$-\frac{1}{2}(\frac{A}{d})^{\frac{3}{2}}s^{\frac{1}{2}} < 0$	$-\frac{3}{2}(sA)^{\frac{3}{2}}d^{-\frac{5}{2}} < 0$	$-(1-s)\frac{1}{2}(\frac{A}{d})^{\frac{3}{2}}s^{\frac{1}{2}} < 0$
∂s	$\frac{1}{2}A^{\frac{3}{2}}(sd)^{-\frac{1}{2}} > 0$	$\frac{3}{2}(\frac{A}{d})^{\frac{3}{2}}s^{\frac{1}{2}} > 0$	$(1-s)\frac{1}{2}A^{\frac{3}{2}}(sd)^{-\frac{1}{2}} - A^{\frac{3}{2}}(\frac{s}{d})^{\frac{1}{2}}$**?**$0$

[10]The text uses stars to indicate steady state values. I am omitting these.

Most of these results are unsurprising. The first row shows the effects of changing A. Suppose that TFP increases, possibly due to improved technology. Any amount of capital and labor now produces more output. As expected, output and consumption both increase. Because there is more output, savings and capital also increase. The second row shows the effects of changing the depreciation rate. If capital wears out more rapidly, it is unsurprising that the steady sate capital stock is reduced. With less capital, there is also less output and consumption. The final column shows the effects of changing the savings rate. As s increases, there is more capital and hence more output. But the final result is less expected. The effect on steady state consumption is ambiguous, it may either increase or decrease.

This final results raises an important point. Good theoretical models often produce ambiguous results.[11] It is not a sign of a bad model.

Because welfare is much more closely tied to consumption than investment, it is important to understand why increasing s can reduce c. Some graphs are helpful.

Graph: Solow

The Solow Model may be graphed using three lines. The depreciation function is a straight line through the origin with slope equal to d. It represents the rate of lost capital. The production function has its usual properties, it starts at the origin, is nearly vertical for low values of k, and becomes flatter as k increases. Finally, the savings/investment function is a scaled down version of the production function, if the latter equals 1, then the production function and savings function are identical. The savings function represents the level of new capital.

[11] I have also found that students hate answering an exam question with "it depends." You can expect some questions like this this term. I will, of course, be interested primarily in your explanation of why the answer is ambiguous.

The steady state is defined by a constant level of capital. This occurs only where new capital and lost capital intersect. To find steady state output, move vertically from the investment function to the production function; the gap between the two is consumption.

Now consider the following exercise. First, graph the model for $s = 0$.

Graph: Solow, $s = 0$

In this case, the investment function always equals zero and thus lies atop the horizontal axis. The steady state occurs when $k = c = y = 0$. Because nothing is saved, there is no capital. Without capital, nothing produced and there is nothing to consume. Equilibrium thus resembles a Mad Max movie.[12] Now suppose that s increases to an intermediate value. The model now resembles our prior graph. Consumption clearly increases as capital and output become positive. Note from our table that evaluating $\frac{\partial c}{\partial s}$ at zero results in an unambiguously positive value. For low levels of savings, saving more increases consumption.

Now suppose that savings increases to $s = 1$.

Graph: Solow, $s = 1$

[12] Yeah, I know this is before your time.

In this case, the investment function lies atop the production function. Consumption, the gap between the two, is clearly zero. Although output is very high (maximized), all of it is saved and there is nothing left for consumption. Note from our table that evaluating $\frac{\partial c}{\partial s}$ at one results in an unambiguously negative value.

It is thus possible to save too much (and have too much output) in this model.[13] We can calculate the optimal (sometimes called the "golden rate") as saving. This requires that we set $\frac{\partial c}{\partial s}$ equal to zero. Doing so yields:

$$s^{gr} = \frac{1}{3} \tag{18}$$

Convergence

So far, all of our results are valid only at the steady state. It is also important toe examine what happens when the model is away from its steady state. To do so, we will employ a technique known as taking a linear approximation around the steady state. The relevant mathematical tool is a first-order Taylor Series approximation. Suppose that we have the following equation:

$$x_{t+1} = \sqrt{x_t} \tag{19}$$

Note that a steady state of this equation is $x = 1$. If the variable ever equals this value, then it will no longer deviate.

The formula for a Taylor Series expansion of the right hand side is:

$$\sqrt{x_t} \approx \sqrt{x} + \frac{\partial \sqrt{x_t}}{\partial x_t}|_x (x_t - x) = 1 + \frac{1}{2}(x_t - x) \tag{20}$$

This technique yields an approximation that is valid if we are "close enough" to the steady state. For some intuition, consider a graph of the right hand side of (19):

Graph: Linear approximation of $\sqrt{x_t}$*:*

[13]Note that I am making a claim about welfare here. Because there are no other factors that affect welfare in this model besides consumption, I think it is reasonable to do so.

We are taking the derivative of this function at a chosen point (the steady state). Our approximation then treats this derivative as constant by extending a vertical line (a tangent) from the steady state. We are then using this tangent line as an approximation to the non-linear line which is too hard to work with directly. Note that if we are close to the steady state, then the tangent line is close to the actual function and our approximation is valid. If we are too far away from the steady state, however, then the approximation becomes dubious.

We can do the same approximation for the left hand side of (19):[14]

$$x_{t+1} \approx x + \frac{\partial x_{t+1}}{\partial x_{t+1}}|_x (x_{t+1} - x) = 1 + (x_{t+1} - x) \tag{21}$$

It then follows that:

$$(x_{t+1} - x) \approx \frac{1}{2}(x_t - x) \tag{22}$$

We now employ a couple of tricks. First, the cool kids typically replace the approximation sign with an equals sign at this point. This is technically incorrect but as long as we keep in mind that we are working with an approximation, it does not cause problems. Second, we can define $\tilde{x}_t = (x_t - x)$ as the deviation of x_t from the steady state. Doing so yields a neater equation:

$$\tilde{x}_{t+1} = \frac{1}{2}\tilde{x}_t \tag{23}$$

Linear approximations are very common in macroeconomics.[15] Many students struggle with them at first and there is often much wailing and gnashing of teeth. But, in most cases, a little practice makes them much more approachable. Don't panic.

We now employ this technique for the Solow Model. Recall that the model can be represented as:

$$k_{t+1} = (1 - d)k_t + sAk_t^{\frac{1}{3}} \tag{24}$$

The first step is to approximate the left hand side:

$$k_{t+1} \approx k + \frac{\partial k_{t+1}}{\partial k_{t+1}}|_k (k_{t+1} - k) = (k_{t+1} - k) = \tilde{k}_{t+1} \tag{25}$$

Now do the same for the right hand side:

[14] Actually, because the left hand side is linear, the approximation is the function itself and we could drop the $\approx$ for a $=$ is we wanted to.

[15] Take it as a compliment that we are doing this. I would not try this at a mediocre school.

$$(1-d)k_t + sAk_t^{\frac{1}{3}} \approx (1-d)k + sAk^{\frac{1}{3}} + \frac{\partial[(1-d)k_t + sAk_t^{\frac{1}{3}}]}{\partial k_t}|_k(k_t - k) = (1-d+\frac{1}{3}sAk^{-\frac{2}{3}})\tilde{k} \quad (26)$$

Examining (24) at the steady state reveals that $k = (1-d)k + sAk^{\frac{1}{3}}$. The constants on both sides of the approximation thus cancel out. We are left with:

$$\tilde{k}_{t+1} = (1 - d + \frac{1}{3}sAk^{-\frac{2}{3}})\tilde{k}_t \quad (27)$$

The final step is to eliminate k using (15):

$$\tilde{k}_{t+1} = (1 - \frac{2}{3}d)\tilde{k}_t \quad (28)$$

Equation (28) shows that the rate of convergence depends only on the depreciation rate. Higher depreciation rates mean that convergence occurs more rapidly. To see this, suppose that $\tilde{k}_t = 1$, implying that the economy initially enjoys extra capital. If depreciation is near one, then per capita capita evolves so that $\tilde{k} = [\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, ...]$. Higher depreciation means that this extra capital wears out rapidly.

Note that the rate of convergence does not depend on s. Although s affects the steady state, it doesn't affect how fast the economy gets there.

We continue to analyze the Solow Model with three experiments.

Experiment #1: Too little capital

Suppose that an economy is initially at its steady state. Now suppose that a war destroys half of the capital stock. Note that as long as s and A, and d are unaffected, the steady state will not change:

Graph:

The war causes capital to fall below its steady state value ($\tilde{k}_t < 0$). Note that the investment function lies above the depreciation function. This implies that more capital is being created than is lost, moving the economy to the right on the graph. Consistent with (28), the economy converges back toward the steady state.

This experiment is helpful in understanding the post World War II experiences of Japan and Germany. Both countries suffered a massive lost of capital as part of their military defeats. In the decades following the war, however, both countries experienced rapid economic growth that could not be matched by the United States. In the context of the Solow Model, they were converging back toward their steady states.

Just as GDP is not welfare, neither is GDP growth. This experiment illustrates that one way to experience sustained higher growth is to blow up half of the factories in the economy. Most people would oppose such a policy.

Experiment #2: Higher savings rate

Now suppose that that the savings rate rate increases. In this case, the steady state capital stock also increases.

Graph:

The increase in the savings rate shifts the investment function upwards. The capital stock then begins to increase because investment exceeds depreciation. The economy thus converges to the new steady state with more output. Note that the effect on consumption is ambiguous.

Experiment #3: A Poverty Trap

Suppose that we wish to use the Solow Model to explain the difference in per capita output between the U.S. and Burundi. One explanation is that the two countries have the same steady state, but are at different points in converging toward that steady state. This, however, is surely not plausible. A more appealing explanation is that differences in A, s, and d explain the difference. A third explanation is that the model may have multiple steady states.

Technically, the Solow Model always has two steady state. The first is the one we have examined in these notes. The second is $k = 0$. But this zero steady state is *unstable*. Note from our original graph, that if k is very small, but positive, then the model will move away from zero and toward the non-zero steady state. The zero steady state is thus of little economic interest.

Now suppose that savings is more complicated. Suppose that, up to some level of subsistence, s is zero, meaning very poor households do not have the means to save at all. Then, for some level of output, s becomes positive. It is possible that the new graph of the Solow Model takes the following shape:[16]

Graph:

Now there are three steady states, including zero. Suppose that the economy begins between the zero and intermediate steady states. Because depreciation exceeds investment, the economy will move toward zero. This is known as a *poverty trap*. Short of an outside infusion of capital, or a change in saving patterns, the economy is unable to escape the zero steady state.

But suppose that the economy is initially above the intermediate steady state. In this case, the

[16]It is also possible that there will be only a zero steady state.

economy will converge toward the high steady state. It is therefore possible for two economies with the same savings pattern, depreciation, and TFP to have very different long run properties.

Additional Results and Extensions

The Solow Model is a valuable introduction to modeling economic growth. It produces useful and empirically supported predictions, while also being simple enough for this class. It does not explain every factor that influences growth. Critics point out that treating human capital as exogenous renders us unable to analyze an important factor of production and the policies that may influence that factor. A different school of thought, known as *endogenous growth* attempts to expand the analysis by making human capital endogenous.

Another criticism is that the Solow Model lacks microfoundations. Households make their savings and consumption decisions mechanically, s is not the result of a utility maximization problem. As a result, equilibrium may not be efficient (as illustrated by having s be too high or too low). It is possible to endogenize the savings decision. Such a task, however, is beyond the scope of this course. The resulting model, however, is efficient.[17] Households always choose s so that utility is maximized.

There are two final and important results to take from the Solow Model. First, most economists interpret (15) as implying that technological progress is the only realistic source of persistent growth. Output can grow faster than technology for a while if, for example, educational attainment is increasing or the savings rate is increasing. But in the very long run, per capita output cannot do much better than grow at the same rate as technology. This helps explain why it appears very hard for a developed economy like the U.S. to exceed about 3% growth in the long run. Second, there isn't a formal role for policy in the model. The model may be used to argue that policymakers should promote more or less saving, or encourage TFP growth, but these are not built into the model itself.

The Solow Model and Barro's Results

We conclude by comparing the Solow Model's theoretical predictions with Barro's empirical results.

1. The Solow Model predicts that increased savings/investment increases aggregate output. Barro finds a similar result.

2. The Solow Model predicts convergence, countries with lower levels of GDP will grow quicker than richer countries. Barro finds a similar result.

[17] The resulting model is known as a Ramsey Model. It is commonly studied in graduate macroeconomics courses.

3. The Solow Model predicts that increased TFP and decreased depreciation increase aggregate output. Barro does not examine these factors, probably because good data are not available. These theoretical results, however, seem obvious.

4. The Solow Model does not model many of the factors that Barro finds are important: education, fertility, institutions, etc. Because of this, it is best to view the Solow Model as a good starting point to study growth. More complex models can add some of these factors.