ECO 270: Growth, The Endogenous Growth Model[1]

The Solow Model offers some important insights into economic growth. But once that model converges to its steady state, the only source of sustained growth is from increased TFP. Explaining why TFP increases is limited in the Solow Model to assuming exogenous technological progress.[2]

We now consider an alternate modeling approach, known as endogenous growth, that further explains TFP. This model was developed by macroeconomist Paul Romer in 1990. Technically, this model endogenizes TFP. The intuition behind the model is that ideas are a type of public good that exhibit nonexcludability and nonrivalry of consumption. For example, suppose that my firm (Vandalay Enterprises) develops a new type of software that increases productivity. Depending on the state of property rights, it may be possible for other firms to immediately copy my innovation and boost their own productivity. Even if patents prevent this from happening immediately, they may be able to imitate my innovation to create a similar version. This is a type of production spillover.[3] My innovation improves the productivity of everyone else in the economy.

The model assumes that the market for ideas is missing. Because I care little about aggregate productivity, I will produce too few ideas. Because markets are incomplete, a free market will be inefficient.

We now formally setup and solve the Romer model. Because it is a more difficult model, we will simplify it by eliminating capital (setting $K = 1$ for all periods). This simplification does not affect the model's main results. As with the Solow Model, we begin by stating some of the model's assumptions:

1. The total amount of labor is $\bar{L}$. This is divided between labor used for production, L_{yt}, and labor used to create new ideas, L_{at}. We can think of the production of ideas as being closely related to research and development. L_{at} thus includes scientists, researchers, and dashing macroeconomists. This assumption yields:

[1] These are undergraduate lecture notes. They do not represent academic work. Expect typos, sloppy formatting, and occasional (possibly stupefying) errors.

[2] Improved policies, such as switching from communism to capitalism, may also increase TFP. But once these opportunities are exhausted, in the very long run, only technological progress remains.

[3] The text and these notes characterize ideas as a public good. It is also common to think of them as an externality.

$$L_{yt} + L_{at} = \bar{L} \tag{1}$$

2. The creation of new ideas is increasing in the existing number of ideas and the amount of labor committed to producing them. The latter is obvious, The former is known as "standing on the shoulders of giants." Existing innovations make it easier to discover new innovations. Newtonian physics, for example, made it easier for Einstein to discover relativity, which made it easier for someone to discover the Snuggie.

3. Ideas are never forgotten. This is generally sensible. While there are occasional examples of technological regression (*e.g.* NASA forgetting how to go the moon, Microsoft Vista being worse than the previous operating system), this is rare. A good assumption need not cover every single anecdote. Together, #2 and #3 may be represented as:

$$A_{t+1} - A_t = \Delta A_{t+1} = \bar{z} A_t L_{at} \tag{2}$$

the parameter $\bar{z}$ allows us to vary the productivity of research and development. As $\bar{z}$ becomes larger, it is easier to create new ideas. If $\bar{z} = 0$, then new ideas are never created.

4. The production function is:

$$Y_t = A_t L_{yt} \tag{3}$$

5. A constant fraction, $\bar{l}$ of labor works in research and development:

$$L_{at} = \bar{l}\bar{L} \tag{4}$$

Note that the model includes both $\bar{l}$ and $\bar{L}$. I assume that this is because Jones hates you and wants to make your life as hard as possible.

This assumption is similar to the Solow Model's assumption that a constant fraction of output is saved. Like that assumption, it is subject to the criticism that it does not result from a microfounded utility maximization problem. Were this a graduate class, we would set up such a problem. But the basic idea is the same so we will choose this simpler approach.

Because $\bar{l}$ is exogenous, it is possible that too much labor is allocated to research development. For example, if $\bar{l} = 1$, then there is no output and technological progress is inefficiently high. Were the share of labor devoted to R&D to be the result of utility maximization, it would necessarily be too low due to the production spillover.

The model thus consists of (1)-(4). Solving the model is somewhat tedious, but not too difficult. It entails substituting these equations into each other while making a few clever algebraic maneuvers. The goal is to represent per capita output as a function of the exogenous variables. Combining (1) and (4) yields $L_{yt} = (1 - \bar{l})\bar{L}$. Inserting this into (3) and dividing by $\bar{L}$ yields:

$$y_t = \frac{Y_t}{\bar{L}} = A_t(1 - \bar{l}) \tag{5}$$

We are thus left with two equations, (2) and (5) and two endogenous variables, Y and A. Begin by re-dating (moving everything back one period) Equation (2) so that $A_t - A_{t-1} = A_{t-1} + \bar{z}L_{at}$. Rearranging and eliminating L_{at} using (4) yields:

$$A_t = (1 + \bar{z}\bar{l}\bar{L})A_{t-1} \tag{6}$$

The growth rate (denoted g in Jones) of TFP is $\bar{z}\bar{l}\bar{L}$. We already mentioned that $\bar{z}$ describes the ease with which new ideas are created. It is not surprising that as developing ideas becomes easier, there is more innovation. Likewise, as $\bar{l}$ or $\bar{L}$ increase, then more labor is dedicated to research and development, and TFP grows faster.

If (6) is true, then it must also be the case that:

$$A_{t-1} = (1 + \bar{z}\bar{l}\bar{L})A_{t-2} \tag{7}$$

Combining the right hand side of (7) with the right hand side of (6) (this is known as iterating backward) yields:

$$A_t = (1 + \bar{z}\bar{l}\bar{L})^2 A_{t-2} \tag{8}$$

We can then repeat this enthralling process t times so that:

$$A_t = (1 + \bar{z}\bar{l}\bar{L})^t A_0 \tag{9}$$

where A_0 is some initial level of TFP. The final step is to use (9) to eliminate A_t from (5). This yields the model's solution:

$$y_t = A_0(1 - \bar{l})(1 + \bar{z}\bar{l}\bar{L})^t \tag{10}$$

Output therefore endogenously grows over time. This is in contrast to the Solow Model where growth eventually requires exogenous changes in TFP. Here it a result of the model.

Another important contrast with the Solow Model is that there is no tendency to converge. If the United States and Burundi begin with different initial values of TFP (A_0), but are otherwise identical, then we expect both economies to grow at the same rate. Burundi's GDP will not catch that of the U.S., and the absolute gap in GDP will widen.

Recall in the Solow Model that per capita output did not depend on population. In this model, as $\bar{L}$ increases, there are more ideas generated and growth speeds up. There is little support for the prediction that more populous nations grow faster. If we interpret $\bar{L}$ as global population, however, then it does appear that global economic growth has increased along with population. This is sensible because new ideas eventually cross political borders.

The Romer Model also yields a broader set of factors to explain dramatic cross country differences in income. Recall the example of the United States and Burundi. TFP in the U.S. is ten times higher than in Burundi. The Solow Model does not go beyond assuming this away as an exogenous difference. This model pins down some factors that may help explain this difference in TFP. For example, perhaps the United States dedicates a higher fraction of labor to research and development than Burundi. This is reflected in a higher value of $\bar{l}$ and (10) shows that this improves growth. We can also interpret $\bar{z}$ as reflecting the quality of each country's institutions. Better courts, universities, etc, may enable the United States to produce ideas more efficiently.

We next consider an increase in $\bar{l}$ so that more resources are dedicated to R&D. Examination of Equation (2) shows that, if this change occurs in period t, there is no effect on A_t. Denoting the initial share of labor dedicated to R&D as $\bar{l}_0$, it follows that:

$$y_t = A_0(1 - \bar{l}_1)(1 + \bar{z}\bar{l}_0\bar{L})^t \tag{11}$$

The final term on the right hand side of (11) captures the growth rate. But in period t, this is unchanged (hence it includes $\bar{l}_0$ instead of $\bar{l}_1$). The middle term represents the share of labor dedicated to production and not R&D. It does decrease immediately (note that it does include $\bar{l}_1$). Output it period t thus unambiguously decreases.

In period $t+1$ and beyond, the growth rate ($\bar{z}\bar{l}_1\bar{L}$) is higher. At some point, this effect will equal and then exceed the effect of less labor for production. Per capita output will thus match and exceed its value for the model without the change to $\bar{l}$. Whether this change is desirable depends largely on how households weight the short term decline in output against the long term increase.

If we define β as the weight that households put on the next period, relative to today, then we can say more about whether an increase in $\bar{l}$ is desirable. If β is very close to zero, the households do not care about the future and it cannot be optimal to increase R&D at the expense of current consumption. If β is very close to one, however, then such a change will be optimal (as long as $\bar{l} < 1$) because the future benefits will eventually outweigh the short term reduction in output.

The following graph captures the effects of the change

Graph:

Note that I am graphing the natural log of output instead of output. This is because if a variable is growing at a constant rate g, then the natural log has a slope equal to g.

To conclude, we have now seen two distinct growth models. Keep in mind that the Solow Model is just one of many classical or neoclassical growth models and this is just one example of many endogenous growth models. Other versions allow for other factors to be explicitly analyzed.