Mortality Shocks and Growth Cycles (or How I Learned to Stop Worrying and Love the Black Death)

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February 6, 2021

Abstract

JEL Classification: .

Keywords: .

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1 Model

The model is embedded in a simple neoclassical growth framework. Households solve a mostly standard optimization problem:

$$Max_{C_t} \quad \sum_{i=0}^{\infty} \beta ln(C_{t+i}) \tag{1}$$

s.t.

$$K_{t+1} = (1-\delta)K_t + N_t^{1-\alpha}K_t^{\alpha} - C_t \tag{2}$$

Optimization then yields a standard Euler Equation.

$$\frac{1}{C_t} = \frac{\beta(1-\delta+\alpha N_t^{1-\alpha}K_t^{\alpha-1})}{C_{t+1}} \tag{3}$$

We now add our first novel assumption; the discount factor, β is simply the probability of being alive in the next period which is just one less the mortality rate.[1] Furthermore, the rate is itself a decreasing function of wealth, which consists exclusively of capital.

$$\beta_t = 1 - l(k_t) \tag{4}$$

There is substantial evidence showing that better economic conditions lead to reduced mortality, although the evidence does not strongly suggest whether the link depends primarily on capital, income, or consumption. Chetty, Stepner, and Abraham (2016) show a that a strong link between higher economic status and life expectancy exists in the contemporary United States. This assumption is also common in the literature on Malthusian economics. Steinmann, Prskawetz, and Feichtinger (1998), for example, develop a model where higher levels of human capital both increase the birth rate and decrease the death rate. Empiricl evidence supports the basic Malthusian connection between economic performance and population growth in pre-industrial economies. Clark and Hamilton (2006) fins that increased wealth led to improved reproductive fitness in England between 1585-1638.

As in the Malthusian literature, we allow population growth to depend on economic status, in this case consumption.[2]

[1]We also considered a version of the model where the discount factor included intrinsic impatience as well as the probability of death. This version produced no noteworthy differences and the present model thus excludes intrinsic impatience.

[2]It makes little difference if population growth instead depends on capital or income.

$$L_{t+1} = (\frac{c_t}{c^*})^\gamma L_t \tag{5}$$

where c^* is the subsistence level of per-capita consumption. It is then straightforward to convert the model to a per-capita basis.

$$k_{t+1}(\frac{c_t}{c^*})^\gamma = (1-\delta)k_t + k_t^\alpha - c_t \tag{6}$$

$$\frac{1}{c_t} = \frac{(1-l(k_t))(\frac{c_t}{c^*})^\gamma(1-\delta+\alpha k_t^{\alpha-1})}{c_{t+1}} \tag{7}$$

The model thus consists of one state variable, k_t, and one control variable, c_t. We now turn our attention to analyzing the different types of solutions that the model may exhibit.

The nature of equilibrium depends primarily on the functional form of $l(k)$, the mortality rate. We first assume that it is bounded between 0 and 1:
Assumption 1: $l(0) < 1$, $l^{'}(k) \leq 0$, and there exists a minimum level of mortality, $\bar{l}$ that is attained at some level of k.

Assumption 1 ensures that mortality cannot equal one, preventing the extinction of the species, that mortality is decreasing in wealth, and that there is some biological minimum on mortality. Later, we show that Assumption ensures the existence of at least one generally stable steady state.

The model has only a few variables to calibrate. We set $\alpha = \frac{1}{3}$, the standard variable for capital's share of income. The depreciation rate is set at 10%, suggesting annual data. For now, we set $gamma = 0$, which implies a constant population. This simplifies the analysis of equilibrium. Later, we isolate how the Malthusian mechanism impacts the model. The only thing that then determines the nature of equilibrium is the mortality rate.

Case II: Multiple Steady States

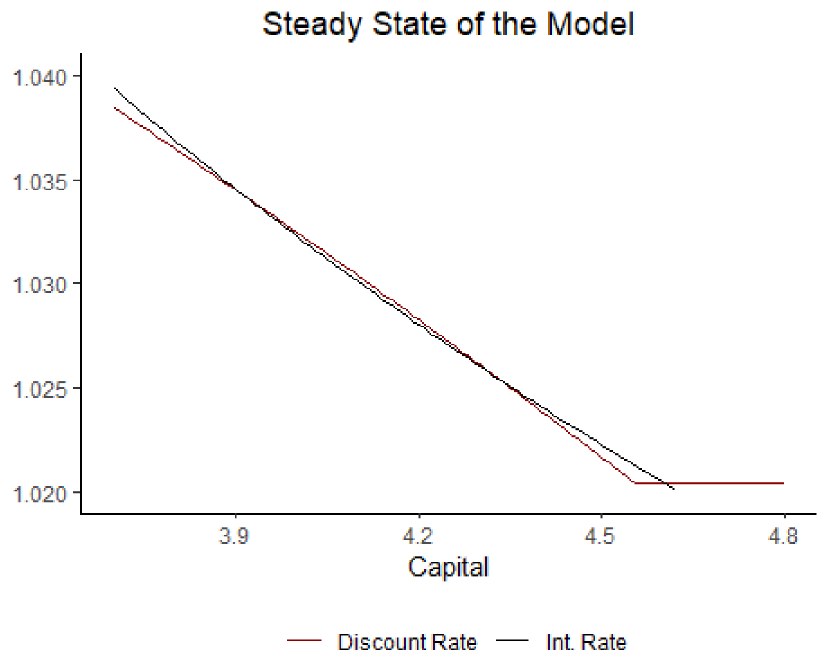
Without population growth, the model's steady state(s) can be evaluated using a single equation. Equation (7) becomes:

$$\frac{1}{1-l(k)} = 1-\delta+\alpha k_t^{\alpha-1} \tag{8}$$

The left hand side of (8) is the inverse of the discount factor while the right hand side is one plus the real interest rate. Assumption 1 ensures that, for low enough values of k, the right hand side is above the left hand side. Our assumption of a minimum mortality rate ensures that there is at least one point where the right hand side crosses from above, which typically results in a stable equilibrium. It is easy, however, to design a mortality function which yields multiple equilibria. The following function yields two stable and one unstable steady state:

$$l(k) = max[0.02, .07 - .0024k^2] \tag{9}$$

Figure 1: Multiple, Stable Steady States, $l(k) = max[0.02, .07 - .0024k^2]$



The local stability of each steady state is evaluated by linearizing the model:

$$X_t = AX_{t-1} + e_t \tag{10}$$

Stability then depends on the eigenvalues of A. If one is outside the unit circle, then there is one saddle condition to pin down the model's only control variable, c_t. If, however, there are two such eigenvalue,s then no solution exists and the steady state is unstable. The model could also have no such eigenvalues, in which case sunspot equilibria may exist . We do not encounter any such cases, however, when analyzing the model.

Table 1 reports the characteristics of each steady state from Figure 1.

The high capital steady state corresponds to higher income, lower interest rates, but less consumption than the low capital steady state. Despite lower consumption, however, the higher

Table 1: Steady State Properties

	Low	Medium	High
k	3.918	4.322	4.606
l(k)	0.33	0.25	0.02
Life Exp.	30	40	50
Stable	Yes	No	Yes
c	0.9888	0.9806	0.9729
r	3.41%	2.56%	2.04%

life expectancy (50 versus 30 years) presumably implies that the high capital steady state is preferable to the low capital steady state.

We now simulate a stochastic version of the model. We add a mortality shock to the model, although similar results can be be obtained through either a preference shock or a productivity shock:

$$\frac{1}{c_t} = \frac{(1 - l(k_t)\mu_t)(\frac{c_t}{c^*})^\gamma(1 - \delta + \alpha k_t^{\alpha-1})}{c_{t+1}} \tag{11}$$

where

$$mu_t = mu_{t-1}^\rho e_t \tag{12}$$

where $ln(e_t)$ is white noise. Numerical exercises show that the boundary between the two stable steady states' basins of attraction is close to the medium capital steady state. We thus simulate the model by linearizing around both the low and high capital steady states. If the capital stock is less than that of the medium capital steady state, then we use the former linear approximation. If not, then we use the latter.

The key result is that the mortality shock can shift the model between steady states. A pandemic that increases mortality can increase capital per worker and shift the model from the low capital steady state to the high capital steady state. Likewise

References

Chetty, R., Stepner, M., and S. Abraham. 2016. “The Association Between Income and Life Expectancy in the United States, 2001-2014.” *Journal of the American Medical Association*, Vol. 315(6): 1750-1766.

Clark, G. and G. Hamilton. 2006. “Survival of the Richest: The Malthusian Mechanism in Pre-Industrial England.” *Journal of Economic History*, Vol. 66(3): 707-736.

Sharp, P., Strulik, H., and J. Weisdorf. 2012. “The Determinates of Income in a Malthusian Equilibrium.” *Journal of Development Economics*, Vol. 97: 112-117.

Steinmann, G., Prskawetz, and G. Feichtinger. 1997. “A Model on tje Escape from the Malthusian Trap.” *Journal of Population Economics*, Vol. 11: 535-550.