Business Cycles: A New Keynesian Model[1]

Warning: This is the most demanding material of the course. Few students will master this topic on the first try. I expect that you will probably need to review this a few times and struggle for a while to nail it down. I strongly encourage you to use the practice problem, these notes, office hours, etc. to do so.

We now begin the development of the course's main business cycle model. Jones covers this material. Chapter 9 is useful background. It is straightforward and so I leave it to you to read on your own. Chapter 11 is the demand side, Chapter 12 is the supply side, and Chapter 13 consists of applications of the model.[2] We will cover only portions of these chapters. My preference would be to explicitly build on the Life Cycle Model. Jones chooses to instead use the Life Cycle model as motivation. Because this material is technical, and I want you to have an additional source, I have reluctantly chosen to defer to Jones's approach, which I still regard as many orders of magnitude better than most other Intermediate Macroeconomics textbooks. On a few occasions, however, I feel compelled to make small changes to the model. I try to make this clear in these notes.

Before proceeding with the formal model, We consider some historical context for the model. We can separate short run macroeconomic theory into two broad traditions.

1. The Classical tradition predates macroeconomics as a subfield of economics. It has historically been very concerned with ensuring that macroeconomics relies on strong microeconomic foundations (*e.g.* utility maximization and profit maximization). As a result, its models bear a strong resemblance to simple supply and demand models, albeit on a much grander scale. The original Classical model fell from grace due to its inability to explain the Great Depression and its 25% U.S. U-3 unemployment rate, which the model predicted must be entirely voluntary unemployment. Since the 1930s, it has seen several neo-classical incarnations. The most recent is the Real Business Cycle Model. This tradition is generally highly skeptical of the ability of monetary and fiscal policies to stabilize the economy.

2. The Keynesian tradition was started by John Maynard Keynes with the publication of the unreadable *The General Theory of Employment, Interest and Money* in 1936. The key

[1]These are undergraduate lecture notes. They do not represent academic work. Expect typos, sloppy formatting, and occasional (possibly stupefying) errors.

[2]Chapter 10 introduces the Great Recession. Read it if you are interested.

assumption of this tradition is that prices (including wages) may be sticky (slow to adjust to changing circumstances). This assumption generally allows monetary and fiscal policies to possibly stabilize the business cycle. The original Keynesian model remained in favor until the 1970s. At this time, poor policy predictions and a lack of microeconomic foundations caused it to fall out of favor. The New Keynesian model attempts to remedy these deficiencies while establishing better microeconomic foundations. It has emerged as the dominant model for short-run macroeconomic policy analysis.

Our model lies in the Keynesian tradition and is best thought of as a simplified New Keynesian model. The New Keynesian model is certainly not the only contemporary model of business cycles, but it does enjoy the most empirical support and is the most influential both among policy makers and within academia.

Our model enjoys decent microeconomic foundations. We will not generally formalize these, but will instead rely on microeconomic intuition, including the Life Cycle Model. To develop the model, we in turn model the markets for goods and services (aggregate demand), labor (aggregate supply), and money.

The Demand Side: The IS/Euler Equation

The IS/Euler Equation represents our modeling of the aggregate markets for ordinary goods and services. It does not encompass markets for either money or labor. We start by making some of our assumptions explicit:

1. The natural rate of output (*aka* potential output), denoted $\bar{Y}_t$, is exogenous. As always, this indicates the level of output that would occur if unemployment equaled its natural rate and capital was fully utilized. Although policy may be able to affect $\bar{Y}_t$, our focus is on explaining short run fluctuations. We thus assume away any effects of policy on the long run.

It is reasonable to think of $\bar{Y}_t$ as the level of output determined by a model of economic growth, such as the Solow or Endogenous Growth models.

2. The long run real interest rate, denoted $\bar{r}$, is also exogenous. This result follows from #1, although we will omit its derivation for time's sake.

The next step is to recall the national income identity:

$$Y_t = C_t + I_t + G_t + EX_t - IM_t \tag{1}$$

Now, more assumptions:

3. Households consume a constant fraction (denoted $\bar{a}_c$) of potential output. Importantly, household consumption depends on potential output instead of actual output. The motivation for this assumption comes from the Life Cycle Model where households try smooth their consumption. They do not like abrupt changes in their consumption habits. They thus track the less volatile $\bar{Y}_t$ rather than Y_t, which is subject to short run fluctuations. Formally:

$$C_t = \bar{a}_c \bar{Y}_t \tag{2}$$

4. Government expenditures equal a constant share (denoted $\bar{a}_g$) of potential output. Formally:

$$g_t = \bar{a}_g \bar{Y}_t \tag{3}$$

5. Exports and imports equal a constant share (denoted $\bar{a}_{ex}$ and $\bar{a}_{im}$) of potential output. Formally:

$$EX_t = \bar{a}_{ex} \bar{Y}_t \tag{4}$$

$$IM_t = \bar{a}_{im} \bar{Y}_t \tag{5}$$

The intuition behind our fourth and fifth assumptions is similar to that of our third. Because government expenditures and imports also provide utility to domestic households, and exports provide utility to foreign households, agents wish to smooth these variables by tying them to the more stable potential output instead of the more volatile actual level of output..

6. The ratio of investment to potential output equals the following:

$$\frac{I_t}{\bar{Y}_t} = \bar{a}_i - \bar{b}(R_t - \bar{r}) \tag{6}$$

This assumption requires more explanation. R_t is the short run real interest rate. For now, it is exogenous. $\bar{a}_i$ represents investment's long run share of potential output. In the long run, $R_t = \bar{r}$, and it follows from (6) that $\frac{I_t}{\bar{Y}_t} = \bar{a}_i$.

In the short run, however, investment depends on the interest rate. When a household wishes to invest in new housing, or when a firm wishes to expand its operations, it typically does so by borrowing through a mortgage or issue of bonds. When interest rates increase, it therefore becomes more expensive to finance investment. We thus expect a utility maximizing household to invest less in new housing and a profit maximizing firm to also invest less. Therefore $\bar{b} > 0$. This parameter represents the sensitivity of investment to short run changes in the interest rate. Values close to zero suggest that investment does not change much in response to movements in interest rates. This is known as the *interest rate channel*.

There is a second basis for this assumption known as the *credit channel*. Borrowers do not actually pay the risk free rate on their loans. They actually pay a risk premium on top of a riskless rate that compensates lenders for the risk and cost of default. The monthly payment on a loan depends on both the size of the loan and the interest rate. Lower riskless rates reduce monthly payments, which reduces default risk, which then reduces risk premiums. This further reduces monthly payments, which reduces default risk... The process repeats itself over and over. Lower riskless rates thus also encourage investment by stabilizing financial markets and thus reducing risk premiums.

Most of the time, risk premiums are fairly small and the credit channel is not too important. During rare financial crises, however, the credit channel often becomes the more important channel and the focus of much monetary policy.

Obtaining the IS/Euler curve requires a few algebraic tricks. First, divide (1) by $\bar{Y}_t$.

$$\frac{Y_t}{\bar{Y}_t} = \frac{C_t + I_t + G_t + EX_t - IM_t}{\bar{Y}_t} \tag{7}$$

Now substitute (2)-(6) into the right hand side of (7):

$$\frac{Y_t}{\bar{Y}_t} = \bar{a}_c + \bar{a}_g + \bar{a}_i + \bar{a}_{ex} - \bar{a}_{im} - \bar{b}(R_t - \bar{r}) \tag{8}$$

Finally, define $\tilde{Y}_t = \frac{Y_t - \bar{Y}_t}{\bar{Y}_t}$. This term represents the *output gap*. It represents the percent deviation of actual output from its natural rate. This is the part of output caused by the business cycle. A recession is a period where the output gap is sufficiently negative. Subtracting 1 from both sides of (8) yields the IS curve:

$$\tilde{Y}_t = \bar{a} - \bar{b}(R_t - \bar{r}) \tag{9}$$

where $\bar{a} = \bar{a}_c + \bar{a}_g + \bar{a}_i + \bar{a}_{ex} - \bar{a}_{im} - 1$.

I now make a change to Jones's model. In the long run (steady state), output must equal potential output. It also follows that an additional dollar of potential output must be completely allocated between consumption, investment, government spending, and net exports. Thus, in the long run $\bar{a}_c + \bar{a}_g + \bar{a}_i + \bar{a}_{ex} - \bar{a}_{im} = 1$, and $\bar{a} = 0$. Because the model is symmetric where negative shocks are the reverse of positive shocks, the average value of $\bar{a}$ must also equal 0, the long run value.

In the short run, however, $\bar{a}$ may deviate from one. It is therefore a random variable. Jones does treat it this way, but he labels it as a constant, which I find confusing. We will thus proceed using the following relationship:

$$\bar{a} = g_t \tag{10}$$

The IS/Euler Equation is thus:

$$\tilde{Y}_t = g_t - \bar{b}(R_t - \bar{r}) \tag{11}$$

We can then interpret g_t as a random shock to aggregate demand that, on average, equal zero. The following includes some events that may be interpreted as positive values of g_t:

1. Increases in government spending above their average. Although it would be sensible to model these as endogenously adjusting to macroeconomic conditions, we assume that Congress just randomly changes these.[3]

2. Changes in households' expectations. Suppose, for example, that households become unusually optimistic about future income. Using the Life Cycle Model for intuition, this could cause them to increase their current consumption.

3. Increases in asset prices. A bubble in stocks or housing, for example, increases households' assets and, using the Life Cycle Model for intuition, incentivizes them to increase their consumption. It would be better to include a formal explanation for these bubbles. This is very challenging and is the subject of much current research. So will we instead just cram bubbles into g_t.

Implications of the IS/Euler Curve:

1. All else equal, a reduction in short-run real interest rates (R_t) increases the output gap (and output). Lower interest rates reduce the cost of financing investment. because investment

[3] On second thought, assuming a stupid Congress might be the most plausible assumption of the semester.

is a part of national income, greater investment increases Y_t. In a graph, this represents a movement along the curve.

This result will have important implications for monetary policy. Most of the time, when the Federal Reserve lowers interest rates, it is trying to exploit this interest rate and credit channels in order to stimulate the economy.

2. All else equal, increases in government spending increase output. Intuitively, government expenditures are part of national income. Increasing g_t thus increases output, provided that interest rates remain unchanged. In a graph, this represents a shift of the curve.

3. Taxes do not affect the output gap. This is an unusual result, most New Keynesian type models do predict that lower taxes is an alternative form of fiscal stimulus, usually less effectively than increasing government spending. The lack of effect results from the assumption that consumption depends only on potential output. If consumption also depends on the output gap, then tax cuts will be effective at influencing output.

Another way of thinking about #3 is that Ricardian Equivalence holds in this model. Alternatively, we can include tax cuts as positive shocks to demand ($g_t > 0$). This does entail taking the model less literally because they are not part of the micro level problem.

The MP Curve

Deriving the monetary policy curve is simple. We need only understand the difference between real interest rates (R_t) and the nominal interest rate (i_t). The nominal interest rate is the market price of borrowing money. If A lends B $100 for one year, and B then pays A back $110 in one year, then $i_t = 10\%$. B is compensating A for delaying his consumption. Part of this compensation is for inflation. $100 in one year may not have the same purchasing power as $100 today. In order to compensate the borrowers, part of the interest rate includes inflation. The real interest rate is the part of the nominal interest rate that excludes inflation. This is known as the *Fisher Equation.*[4]

$$i_t = R_t + \pi_t \tag{12}$$

The monetary authority (the Federal Reserve in the United States) is able to set the nominal interest rate (i_t) through open market operations. We assume that it is able to successfully hit its target. Having made this assumption, we rename the Fisher Equation the MP curve.

[4]More generally, the real interest rate excludes expected inflation. Our model, however, substitutes actual inflation for simplicity.

IS-MP

With one temporary assumption, we can combine the IS and MP curves to create an underwhelming model of business cycles. We now briefly assume that inflation is exogenous. This assumption may be reasonable under the following conditions:

i. Recent inflation has been stable so that firms are slow to notice a change in π_t.

ii. The analysis is limited to the very short run so that agents in the economy do not have time to respond to changes in policy.

Combining (9) and (12) yields:

$$\tilde{Y}_t = g_t - \bar{b}(i_t - \pi_t - \bar{r}) \tag{13}$$

Equation (13) demonstrates two possible ways for policymakers to affect output. First, they may increase government expenditures as part of expansionary fiscal policy. Second, the monetary authority may lower i_t as part of expansionary monetary policy. Likewise, contractionary policy may lower $\bar{a}_g$ or increase i_t. These policies will be effective as long as π_t will not be affected.

Example: Policy Response to a Demand Shock

Suppose that aggregate demand is reduced. This may result from a variety of events such as reduced government expenditures, a decline in household optimism about average future income, or the bursting of an asset pricing bubble (perhaps a housing bubble) that reduces lifetime wealth. Mathematically, $g_t < 0$.

Because π_t is exogenous, the Central Bank can offset this decrease in aggregate demand by lowering nominal interest rates, i_t. Real interest rates $i_t - \pi_t$ fall by the same amount. As long as the Central Bank does not run into the zero lower bound on interest rates that requires that $i_t > 0$, it can completely offset the effect on output.

Graph: Demand Shock and Policy Response: R on vertical, $\tilde{y}$ on horizontal

This is not a very good model in my opinion. My main objection is to assumption #6. While I do believe that investment responds to changes in interest rates, it takes time for firms and households to change their investment decisions, so that this effect occurs only with a lag. Empirical evidence suggests a lag of about 18 months for the full effect of monetary policy to kick in. So the IS-MP framework to work, the time frame has to be short enough for inflation to be unaffected but long enough for monetary policy to affect output. The model is therefore logically inconsistent.

It does, however, become a much better model if we add a New Keynesian Phillips Curve.[5]

New Keynesian Phillips Curve

The New Keynesian Phillips Curve represents the production side of the economy. We assume the following:

1. The production side of the economy is not perfectly competitive. Instead, it features monopolistic competition where all suppliers have some market power.

2. Firms must choose their price for each period without knowing the prices of their competitors. This is an example of *sticky prices*, a hallmark of the Keynesian tradition.

Both #1 and #2 represent departures from complete and competitive markets. #1 is a departure from perfect competition. #2 is an example of imperfect information. As a result,

[5] Jones usually refers to this as simply a Phillips Curve.

equilibrium will generally not be efficient and it may be possible for policy to result in Pareto improvements.

It is possible to mathematically show that a profit maximizing firm will set their price so that:[6]

$$\pi_t = \pi_t^e + \bar{v}\tilde{Y}_t \tag{14}$$

Equation (14) shows that firms care about two variables when setting their prices. First, they want to keep up with their competitors and input suppliers. If Pepsi expects their wage and capital bills to increase by 5%, and also expects Coke to increase its price by 5%, then it also is tempted to choose to increase its price by 5%. Second, firms also care about demand conditions. When $\tilde{Y}_t$ is low, the demand for most products is depressed. A profit maximizing supplier responds to lower demand by reducing their price. The term $\bar{v}$ represents the sensitivity of producers to potential output.

We need one more assumption to finalize the model. This assumption sets:

$$\pi_t^e = \pi_{t-1} \tag{15}$$

Equation (15) assumes that firms expect future inflation to equal its most recent value. This is a reasonable assumption as long as the level of inflation has been fairly stable. Suppose, for example, that inflation has equaled 4% forever. In this case, it is reasonable to expect a rational firm to expect inflation to equal 4% going forward. Suppose, however that inflation has equaled 0%, 4%, 8%, 12%... In this case, a rational firm would probably come to expect increasing inflation.

The model is thus only valid for sufficiently stable levels of inflation. Results derived from policies or events that lead to large changes in inflation should be taken with a grain of salt. Combining (14) and (15) yields the New Keynesian Phillips Curve:

$$\Delta\pi_t = \bar{v}\tilde{Y}_t \tag{16}$$

We can make one addition to the New Keynesian Phillips Curve. We do not pretend that our model includes all the factors that determine inflation. By adding a random shock to (16) we can analyze the effect of other things that affect the macroeconomy:

[6]It takes me about 3 hours of class time to show this. Just trust me on this one.

$$\Delta\pi_t = \bar{v}\tilde{Y}_t + u_t \tag{17}$$

I have made a notational change to Jones. Because supply shocks are a random variable, I am denoting them using u_t instead of $\bar{o}$.

Higher values of u_t represent random events that put upward pressure on inflation. A classic example of such a shock is an increase in energy prices. Other events that increase the cost of production may also be captured by increasing u_t.

Monetary Policy

In reality, the FOMC meets occasionally to determine their targeted short term interest rate. To approximate their behavior, we can assume a monetary rule:

$$i_t = \bar{\pi} + \bar{m}(\pi_t - \bar{\pi}) \tag{18}$$

Jones uses a rule where the Central Bank chooses the real interest rate. I can't live with this. **So I am changing the model so that the Central Bank chooses the nominal interest rate in a similar way**.

In this setup, the monetary authority chooses two parameters, The first is its target (long run) level of inflation, denoted $\bar{\pi}$. In the United States, this is 2%.[7] The second is $\bar{m}$, the responsiveness of monetary policy to inflation. Higher values of $\bar{m}$ suggest that the monetary authority is more aggressive at raising interest rates in response to inflation, thus putting downward pressure on inflation.

Combining (18) with (13) yields:

$$\tilde{Y}_t = g_t - \bar{b}((\bar{m} - 1)(\pi_t - \bar{\pi}) - \bar{r}) \tag{19}$$

The model thus consist only of (17) and (19). The former equation is known as *aggregate demand*, the latter is known as *aggregate supply*.

Major Results

Result #1: Monetary policy can only keep output above potential output in the long run if inflation is always increasing.

[7]For a long time, the Fed avoided clearly articulating their target.

To analyze the long run, we examine the model's steady state. To solve for the model's steady state, we set the model's random shocks (u_t and g_t) to their mean of zero and drop all subscripts. Doing so for (17) yields:

$$\Delta\pi = \bar{v}\tilde{Y} \tag{20}$$

So if $\tilde{Y} > 0$, then $\Delta\pi > 0$. This implies ever increasing inflation.

Most macroeconomists would take this result one step further so that monetary policy cannot affect output in the long run at all. There are two arguments. First, suppose that $\Delta\pi_t > 0$ for all time periods. Eventually, π_t would become so large that the monetary authority would have to abandon whatever policy achieved this outcome. Second, our assumption that $\pi_t^e = \pi_{t-1}$ becomes highly dubious if inflation is always increasing. Replacing this assumption with more sophisticated expectations formation shows that monetary policy cannot affect output in the long run.

Result #2: Monetary policy does determine the long run rate of inflation.

Consider the model's steady state where $\Delta\pi = 0$. Evaluating (19) at the steady state yields:

$$\pi = \bar{\pi} \tag{21}$$

The Central Bank can thus choose the steady state value of inflation, it simply equals its target. The Federal Reserve has chosen a 2% target. This choice is significantly motivated by its desire to avoid deflation.

We can use (21) to simplify the aggregate demand equation. Note that $\bar{r} = 0$. Inserting this into (19) yields:

$$\tilde{Y}_t = g_t - b(m-1)(\pi_t - \bar{\pi}) \tag{22}$$

Result #3. When supply shocks occur (represented by u_t), the monetary authority faces a short run tradeoff between stabilizing output and higher inflation. Its choice of m determines the nature of this tradeoff.

Suppose that the economy starts at $\bar{\pi}$ and $g_t = \tilde{Y} = 0$. Then a one time adverse supply shock ($u_t > 0$) affects the economy. This may represent higher energy prices or other factors that make production less efficient. The aggregate demand curve is downward sloping with intercept equal to $\bar{\pi}$ and slope equal to $-\frac{1}{\bar{b}(\bar{m}-1)}$.

The aggregate supply curve has intercept equal to $u_t + \bar{\pi}$ and slope equal to $\bar{v}$.

Suppose that the monetary authority chooses a very large value of m. Notice that the aggregate demand curve is now flat. The increase in u_t shifts the aggregate supply curve to the left.

Graph: AS/AD (π_t on vertical and $\tilde{Y}_t$ on horizontal). High m.

If the Fed chooses to aggressively raise interest rates, then the effect of the shock is entirely on output, which decreases below its natural rate. Suppose, however, that the monetary authority sets $m = 1$. The aggregate demand curve is now vertical.

Graph: AS/AD (π_t on vertical and $\tilde{Y}_t$ on horizontal). $m = 1$.

Now, the effect of the shock is limited to higher inflation. By choosing various levels of m, monetary policy may alter the balance of higher inflation and lower output in response to adverse shocks. It cannot, however, fully insulate the economy from the combination of higher inflation and lower output.

Result #4. For small shocks to aggregate demand, monetary policy can completely offset their effects.

Consider the following appended monetary policy rule:

$$i_t = \bar{\pi} + \bar{m}(\pi_t - \bar{\pi}) - \phi g_t \tag{23}$$

Suppose that the Central Bank sets $\phi = b^{-1}$. Inserting (23), with this value of ϕ into (13) yields:

$$\tilde{Y}_t = -b(m-1)(\pi_t - \bar{\pi}) \tag{24}$$

Crucially, g_t has been eliminated from the model.

Result #5. Monetary policy cannot offset large negative reductions ($g_t << 0$) to aggregate demand. Nominal interest rates cannot be less than zero (no lender would ever issue such a loan because they are better off putting their money under a mattress). A large enough demand shock may push interest rates to near zero, eliminating the ability to further reduce interest rates. This situation is known as a *liquidity trap*. In this case, the aggregate demand curve shifts to the left and output is reduced.

Graph: AS/AD (π_t on vertical and $\tilde{Y}_t$ on horizontal). Less aggregate demand.

Notable examples of liquidity traps include the Great Depression, Japan for the last 5+ years, and the U.S as of breakfast this morning.[8] Macroeconomists are usually skeptical of fiscal policy as a way to stabilize the economy. In a liquidity trap, however, expansionary fiscal policy and other unusual measures may be needed because ordinary monetary policy is no longer effective.

Result #6: Disinflation is costly. Disinflation refers to a reduction in inflation. This is easily confused with deflation which is a fall in the price level. Suppose that the economy is at its steady state with an initial steady state level of inflation, $\bar{\pi}_0$. Now suppose that the Central Bank decides that it wants to move the economy to a lower inflation target, $\bar{\pi}_1$. Re-writing aggregate supply and aggregate demand for these conditions yields:

$$\pi_t - \bar{\pi}_0 = \tilde{Y}_t \tag{25}$$

$$\tilde{Y}_t = -b(m-1)(\pi_t - \bar{\pi}_1) \tag{26}$$

Inserting (25) into (26) and re-arranging yields:

$$\tilde{Y}_t = \frac{b(m-1)(\bar{\pi}_1 - \bar{\pi}_0)}{1 + b(m-1)} \tag{27}$$

Decreasing the inflation target implies that $(\bar{\pi}_1 - \bar{\pi}_0) < 0$. It follows from (27) that $\tilde{Y}_t < 0$. Eventually, the economy moves to the new, low inflation, steady state. But during the transition, output falls below potential output.

The most popular example of disinflation is the Volker Recession. In the late 1970's, U.S. annual inflation exceeded 10%. To bring this level down, Federal Reserve Chairman Paul Volker instituted contractionary monetary policy. It worked and inflation has not been a significant problem since. During the disinflationary period, however, a sharp recession ensued with unemployment rising above 10%. This used to be known as the worst recession since the Great Depression.[9]

Other New Keynesian Results

[8] Our definition of a liquidity trap focuses only on short run rates. If longer term rates are significantly positive and if the Fed can affect them, then this definition is debatable.

[9] I can't remember why it no longer enjoys that title.

Our model is just one version from a vast literature. Without any formal modeling, here are some other results that come from this literature:

1. Optimal monetary policy (that which maximizes household utility) often focuses on stabilizing prices. This is because the major source of inefficiency in this literature is sticky prices. By stabilizing prices ($\pi_t = 0$), policy makes the fact that some prices cannot change irrelevant.

2. Often, the best way to stabilize output and unemployment is to stabilize prices.

3. Versions which assume sticky wages enjoy better empirical fit than versions that assume sticky goods market prices.

4. Monetary policy should not pay much attention to asset (housing or stocks) prices. This is because these prices are usually assumed to adjust efficiently to changing economic conditions.