LCM and RBC: Key to Practice Problems

1. Using the Life-Cycle Model as developed in class, discuss the effects on consumption of the following changes:

a. Such a change increases the number of periods over which consumption is smoothed without affecting lifetime wealth. Consumption per period thus declines. Intuitively, households must save for a longer retirement.

b. In this case, lifetime wealth increases. Consumption does as well. This occurs even for households who experience no immediate change in their income. They increase their consumption because there is less need to save for retirement.

2. This changes the marginal utility of consumption to $\frac{\partial u(C_t)}{\partial C_t} = C_t^{-\sigma}$. The analysis from class is otherwise unchanged.

i. Suppose that a household increases its current consumption by one (very small) unit. Doing so increases its utility by $C_t^{-\sigma}$ in period t.

ii. By reducing its consumption in period t, the households wealth in period $t+1$ is reduced by $(1 + r_t)$ units.

iii. Suppose that the household then expects to reduce its consumption in period $t + 1$ by $(1 + r_t)$ units. Doing so will leave its wealth in period $t + 2$ and beyond unchanged from the baseline.

iv. The expected lost utility from #3 is $(1 + r_t)C_{t+1}^{-\sigma}$. Because the household discounts, however, this must be multiplied by β.

v. If the benefits of this exercise exceed its costs, then the household, by definition, cannot me maximizing its lifetime utility. Likewise, if the costs exceed the benefits then the household could increase its utility be reducing C_t. So it must be true that the costs equal the benefits. This yields the *Euler Equation*:

$$C_t^{-\sigma} = E_t \left[\beta(1 + r_t)C_{t+1}^{-\sigma}\right] \tag{1}$$

3. Equation (1) may be rewritten so that:

$$E_t[\frac{C_{t+1}}{C_t}] = (\beta(1+r_t))^{\frac{1}{\sigma}} \tag{2}$$

where perfect consumption smoothing is indicated by a left hand side equal to one. Now suppose that r_t increases. As $\sigma \to \infty$, the right hand side always equals one. The change in r_t has no effect on consumption. Higher values of σ thus result in more consumption smoothing.

4. The marginal utility of consumption now equals $2C_t$. You may be tempted to simply insert this result into the steps for #2. This would be incorrect, however. This utility function yields an increasing marginal product of labor. As a result, households no longer wish to smooth their consumption. They will instead choose to consume all of their lifetime wealth in a single period.

5. When a negative productivity shocks occurs, the marginal product of labor, and hence the wage, falls. Households thus rationally choose to supply less labor which yields a bigger reduction of output. When labor supply is perfectly inelastic, however, this effect does not occur and the effect on output is thus minimized.

6. They are bad in the sense that it would be better had the negative productivity shock not occurred. Recessions, however, are efficient responses to such shocks.

7. Involuntary unemployment always equals zero in this model.