Lecture Notes on

MONEY, BANKING, AND FINANCIAL MARKETS

Peter N. Ireland
Department of Economics
Boston College

irelandp@bc.edu

http://www2.bc.edu/~irelandp/ec261.html

Chapter 7: A Model of Stock Prices

1. The Dividend Valuation Model
2. Key Assumptions and Questions
3. The Gordon Growth Model
4. Example: Valuing the Dow Jones Industrial Average

Mishkin's Chapter 7 presents a model of stock prices called the dividend valuation model.

These notes show how this model can be derived by comparing the stream of future dividends paid by a stock to the stream of cash flows paid by a portfolio, or combination, of discount bonds.

Deriving the dividend valuation model in detail this way allows us to see more clearly the key assumptions that are built into the model and to identify the key questions we need to answer in order to apply the model in practice.

When combined with an additional set of assumptions, the dividend valuation model can be used to obtain a version of the Gordon growth model, a famous and still widely-used model of stock prices.

After deriving the Gordon growth model, we can use it to ask whether stocks prices—as measured by the Dow Jones Industrial Average—were too high or too low at the end of 2002.

1 The Dividend Valuation Model

Equity = a contractual agreement representing claims to a share in the income and assets of a business.

Common stock is the principal form of equity.

Owners of common stock receive a bundle of rights, which include:

The right to vote on issues that are most important to the corporation.

The right to be the corporation's residual claimant: stockholders receive whatever funds remain after holders of debt are paid.

Stockholders receive these funds in the form of dividend payments, which are authorized by the firm's board of directors and typically paid quarterly.

Consider a share of stock that sells for price P_t today (at time t).

Recall that equities have no maturity date.

Consistent with this fact, suppose that each share of stock pays dividends D_{t+1}, D_{t+2}, D_{t+3},... starting next period (time $t+1$) and continuing out into the infinite future.

How does the price P_t of the stock today depend on this stream of future dividends, or cash flows?

The dividend valuation model shows us how to answer this question.

Step one in deriving the dividend valuation model is to recognize the following fact:

Fact: The stream of dividends D_{t+1}, D_{t+2}, D_{t+3},... paid by the stock can be replicated by a portfolio, or combination, of discount, or zero-coupon, bonds having different maturities.

To see why this fact must be true, consider the following investment strategy today, at time t:

Buy a one-period discount bond with face value D_{t+1}.

Buy a two-period discount bond with face value D_{t+2}.

Buy a three-period discount bond with face value D_{t+3}.

And so on, out into the infinite future.

This portfolio of discount bonds pays off:

D_{t+1} when the one-period bond matures at time $t+1$.

D_{t+2} when the two-period bond matures at time $t+2$.

D_{t+3} when the three-period bond matures at time $t+3$.

And so on, out into the infinite future.

The portfolio of discount bonds therefore replicates the stream of dividends paid by the stock.

Next, let's ask: what are today's (time t) prices of all of these discount bonds?

To answer this question, let

$$i_{1t} = \text{yield to maturity on the one-period discount bond}$$

$$i_{2t} = \text{annualized yield to maturity on the two-period discount bond}$$

$$i_{3t} = \text{annualized yield to maturity on the three-period discount bond}$$

and so on, out into the infinite future.

Then one-period discount bond has price

$$Q_{t+1} = \frac{D_{t+1}}{1+i_{1t}}$$

at time t.

The two-period discount bond has price

$$Q_{t+2} = \frac{D_{t+2}}{(1+i_{2t})^2}.$$

The three-period discount bond has price

$$Q_{t+3} = \frac{D_{t+3}}{(1+i_{3t})^3}.$$

And so on, out into the infinite future.

Finally, note that the price of the stock must be equal to the total cost of this portfolio of discount bonds, so that

$$P_t = Q_{t+1} + Q_{t+2} + Q_{t+3} + ... \tag{1}$$

To see why this must be the case, suppose instead that the stock price is greater than the cost of the portfolio of bonds, so that

$$P_t > Q_{t+1} + Q_{t+2} + Q_{t+3} + ...$$

Since the stock and the portfolio of bonds both pay off the same stream of future cash flows D_{t+1}, D_{t+2}, D_{t+3},..., in this case it would make sense for investors to sell the stock and buy the portfolio of bonds.

The stock price would then fall and the bond prices would then rise until

$$P_t = Q_{t+1} + Q_{t+2} + Q_{t+3} + ... \tag{1}$$

as required.

Suppose, on the other hand, that the stock price is less than the cost of the portfolio of bonds, so that

$$P_t < Q_{t+1} + Q_{t+2} + Q_{t+3} + ...$$

Since the stock and the portfolio of bonds both pay off the same stream of future cash flows D_{t+1}, D_{t+2}, D_{t+3},..., in this case it would make sense for investors to buy the stock and sell the portfolio of bonds.

The stock price would then rise and the bond prices would then fall until

$$P_t = Q_{t+1} + Q_{t+2} + Q_{t+3} + ... \tag{1}$$

as required.

Now, let's rewrite equation (1) using the expressions for the bond prices that we derived earlier:

$$P_t = Q_{t+1} + Q_{t+2} + Q_{t+3} + ... \tag{1}$$

is equivalent to

$$P_t = \frac{D_{t+1}}{1 + i_{1t}} + \frac{D_{t+2}}{(1 + i_{2t})^2} + \frac{D_{t+3}}{(1 + i_{3t})^3} + ...$$

or, more compactly,

$$P_t = \sum_{j=1}^{\infty} \frac{D_{t+j}}{(1 + i_{jt})^j}. \tag{2}$$

Equation (2) summarizes the dividend valuation model.

Note that the expression on the right-hand side of equation (2) measures the present value of all future dividends paid by the stock.

Hence, in words, the dividend valuation model says that the price of a share of stock should equal the present value of all future dividends paid by that share of stock.

2 Key Assumptions and Questions

The dividend valuation model tells us that the price of a share of stock should equal the present value of all future dividends paid by that share of stock, that is,

$$P_t = \sum_{j=1}^{\infty} \frac{D_{t+j}}{(1 + i_{jt})^j}. \tag{2}$$

One key assumption that underlies the derivation of equation (2) is that a portfolio of discount bonds exists that exactly replicates the stream of dividends, or cash flows, D_{t+1}, D_{t+2}, D_{t+3},... paid by the stock.

In practice, these future dividends are not known with certainty, that is, the stock is a risky asset.

Hence, the discount bonds used in our derivation of (2) should probably not be default-free.

According to the loanable funds framework, when the riskiness of bonds increases, the price of bonds decreases, and hence the interest rate increases.

Hence, the interest rates used in (2) should be higher than the interest rates on US government bonds.

But how do we know exactly what interest rates i_{1t}, i_{2t}, i_{3t},... to use in applying the dividend valuation model?

And how do we forecast the future dividends D_{t+1}, D_{t+2}, D_{t+3},...?

Security analysts and other financial market participants work hard to find answers to these questions.

But the answers are seldom, if ever, clear-cut.

3 The Gordon Growth Model

The Gordon growth model, named after its inventor Myron Gordon, makes the dividend valuation model easier to use by combining it with three additional assumptions:

1. Dividends are growing at the constant rate g, so that

$$D_{t+1} = (1+g)D_t,$$

$$D_{t+2} = (1+g)D_{t+1} = (1+g)(1+g)D_t = (1+g)^2 D_t,$$
$$D_{t+3} = (1+g)D_{t+2} = (1+g)(1+g)^2 D_t = (1+g)^3 D_t,$$

or, more generally,

$$D_{t+j} = (1+g)^j D_t$$

for all $j = 0, 1, 2, ...$

2. The interest rates i_{1t}, i_{2t}, i_{3t},... are constant, so that

$$1 + i_{jt} = 1 + k,$$

where

$$k = \text{required return on equity.}$$

Remember that since the stock is a risky asset, the required return k should generally be greater than the interest rate on US Government bonds.

3. The required return on equity exceeds the dividend growth rate, so that

$$k > g.$$

This assumption is needed for technical reasons, as we'll see below.

To derive the Gordon growth model, combine these assumptions with equation (2) from before:

$$P_t = \sum_{j=1}^{\infty} \frac{D_{t+j}}{(1 + i_{t+j})^j} \tag{2}$$

$$P_t = \sum_{j=1}^{\infty} \frac{(1+g)^j D_t}{(1+k)^j}$$

$$P_t = \left[\sum_{j=1}^{\infty} \frac{(1+g)^j}{(1+k)^j} \right] D_t$$

$$P_t = \left[\sum_{j=1}^{\infty} \left(\frac{1+g}{1+k} \right)^j \right] D_t. \tag{3}$$

Equation (3) is still rather complicated, but it can be simplified using the following fact:

Fact: If $k > g$,

$$\sum_{j=1}^{\infty} \left(\frac{1+g}{1+k} \right)^j = \frac{1+g}{k-g}. \tag{4}$$

Proving this fact requires some algebra. To see why is must be true, start with

$$\sum_{j=1}^{\infty} \left(\frac{1+g}{1+k} \right)^j = \frac{1+g}{k-g}$$

and multiply both sides by $1-\frac{1+g}{1+k}$ to obtain

$$\left(1-\frac{1+g}{1+k}\right)\sum_{j=1}^{\infty}\left(\frac{1+g}{1+k}\right)^{j}=\left(1-\frac{1+g}{1+k}\right)\frac{1+g}{k-g}$$

or

$$\sum_{j=1}^{\infty}\left(\frac{1+g}{1+k}\right)^{j}-\sum_{j=1}^{\infty}\left(\frac{1+g}{1+k}\right)^{j+1}=\left(\frac{1+k}{1+k}-\frac{1+g}{1+k}\right)\frac{1+g}{k-g}$$

or

$$\begin{aligned}&\left(\frac{1+g}{1+k}\right)+\left(\frac{1+g}{1+k}\right)^{2}+\left(\frac{1+g}{1+k}\right)^{3}+\ldots\\&-\left(\frac{1+g}{1+k}\right)^{2}-\left(\frac{1+g}{1+k}\right)^{3}-\left(\frac{1+g}{1+k}\right)^{4}-\ldots\\=&\left(\frac{k-g}{1+k}\right)\frac{1+g}{k-g}\end{aligned}$$

or

$$\frac{1+g}{1+k}=\frac{1+g}{1+k},$$

all of which implies that the two sides of the original equation (4) must be equal.

Since (4) only holds true if $k>g$, we need Gordon's third assumption to make the theory work.

Using equation (4), equation (3) becomes

$$P_t=\left(\frac{1+g}{k-g}\right)D_t. \tag{5}$$

Equation (5) allows us to determine today's (time t) stock price P_t based on:

1. Today's (time t) dividend D_t, which is presumably known.
2. The constant dividend growth rate g, which must be estimated or assumed.
3. The required return on equity k, which must also be estimated or assumed.

Before, moving on, let's ask if equation (5) makes sense. It implies that today's stock price P_t will be higher if:

1. Today's dividend D_t is larger.
2. The growth rate of future dividends g is larger.

3. The required return of equity is k is smaller.

The first two of these implications seem sensible: of course, if the dividend is larger, or growing at a faster rate, the stock price should be higher.

But what about the third implication: that P_t is higher when k is smaller?

Recall that k, the required return on equity, is assumed to be equal to the constant yield to maturity on a portfolio of discount bonds with the same risk characteristics as the stock.

The loanable funds framework implies that if the bonds become less risky, the interest rate will fall.

Hence, a smaller value of k corresponds to a lower level of risk.

So this third implication makes sense too: if the stock is less risky, the stock price should be higher.

4 Example: Valuing the Dow Jones Industrial Average

The Dow Jones Industrial Average (DJIA) is an index, or average, of stock prices for 30 of the largest US corporations: Alcoa, Altria, American Express, AT&T, Boeing, Caterpillar, CitiGroup, Coca-Cola, Disney, DuPont, Eastman Kodak, Exxon-Mobil, General Electric, General Motors, Hewlett-Packard, Home Depot, Honeywell, IBM, Intel, International Paper, JP Morgan, Johnson & Johnson, McDonalds, Merck, Microsoft, Proctor & Gamble, SBC Communications, 3M, United Technologies, and Wal-Mart.

Hence, the DJIA can also be regarded as the price of a portfolio that contains these 30 stocks.

There are other widely-cited stock indices: for example, the Standard and Poor's (S&P) 500. But the DJIA is probably the most famous.

Let's apply the Gordon growth model to value the DJIA at the end of 2002.

To start, note that the dividends paid by the stocks in the DJIA during 2002 equaled \$189.70. This fact dictates a setting of

$$D_t = 189.70.$$

Next, let's make an assumption about g, the growth rate of future dividends, by taking a look at the growth rate of dividends in the past.

Dividends paid by the stocks in the DJIA during 1992 equaled $100.70.

Hence, over the ten years from 1992 through 2002, dividends grew at the average annual rate of 6.54%.

Dividends paid by the stocks in the DJIA during 1982 equaled $54.10.

Hence, over the twenty years from 1982 through 2002, dividends grew at the average annual rate of 6.47%.

Based on these figures, it seems reasonable to expect that dividends will continue to grow at an annual rate of about 6.5%.

This assumption dictates a choice of $g = 0.065$.

Finally, let's make an assumption about k, the required return on equity.

We've noted before that k should be higher than the interest rate on US government bonds.

At the end of 2002, the yield to maturity on the 10-year US Treasury note was 3.83%.

So k should definitely be bigger than 3.83%.

At the end of 2002, the yield on corporate bonds with Moody's highest rating of Aaa was 6.09%.

But equityholders are residual claimants, meaning that bondholders get paid first. And, besides, not all of the companies in the DJIA have Aaa bond ratings. So the stocks in the DJIA are probably more risky than Aaa bonds.

Hence, k should probably be bigger than 6.09% as well.

Based on these observations, let's choose a value of k equal to 8.5%, or $k = 0.085$.

Now let plug our choices of $D_t = 189.7$, $g = 0.065$, and $k = 0.085$ into equation (5):

$$P_t = \left(\frac{1+g}{k-g}\right) D_t \tag{5}$$

$$P_t = \left(\frac{1+0.065}{0.085-0.065}\right) 189.7$$

$$P_t = \left(\frac{1.065}{0.02}\right) 189.7 = 10101.53.$$

Hence, under our assumptions, the Gordon growth model implies that the DJIA should have been traded at 10,101.53 at the end of 2002.

In fact, the DJIA stood at 8,341.63 at the end of 2002.

Under our assumptions, the Gordon growth model implies that stocks were undervalued by over 20%!

But suppose, on the other hand, that stocks are more risky than we've initially assumed, so that a higher value of k is called for.

In particular, let's redo our calculations assuming that k equals 9%, or $k = 0.09$, instead of 8.5%:

$$P_t = \left(\frac{1+g}{k-g}\right) D_t \tag{5}$$

$$P_t = \left(\frac{1+0.065}{0.090-0.065}\right) 189.7$$

$$P_t = \left(\frac{1.065}{0.025}\right) 189.7 = 8081.22.$$

Now, under our new assumption about risk, the model implies that stocks were slightly overvalued instead.

This example illustrates that Gordon's model can be useful in translating assumptions about future dividend growth and risk into implications for stock prices.

But the model's implications are only as reliable as the assumptions that we feed into it.