

# Housing and Endogenous Default\*

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## Abstract

We examine a New Keynesian model with housing where default occurs if housing prices are sufficiently low. If borrowers default, they may lose access to credit and housing markets. Default is not simply a symptom of economic downturns but instead causes discrete drops to aggregate consumption and that of lenders, and increases to borrowers' consumption. It also causes a missallocation of housing that amplifies the initial decline in housing prices. The effects on consumption often peak immediately before default occurs.

*JEL Classification:* E21, E22, E32, E51.

*Keywords:* financial market frictions, credit default, housing, learning.

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# 1 Introduction

In a recent speech given by the former Federal Reserve Bank Chairman Ben Bernanke, he claims:<sup>1</sup>

The multi-year boom and bust in housing prices of the past decade, together with the sharp increase in mortgage delinquencies and defaults that followed, were among the principal causes of the financial crisis and the ensuing deep recession—a recession that cost some 8 million jobs.

Bernanke suggests that mortgage default is not simply a result of economic downturns, but that default itself can exacerbate a downturn. Despite this sentiment and empirical support (see Subsection 2.1), the recent macroeconomic literature largely ignores the effects of default, instead focusing on cases where the threat of default matters, but default never actually occurs.<sup>2</sup> This paper fills this gap. When the economy enters its default region (where all borrowers default), the model exhibits discrete drops in aggregate and lenders' consumption, increases to borrowers consumption, and the decline in housing prices that caused default is dramatically amplified. As default risk builds, impatient households (borrowers) increase their consumption and owner-occupied housing while patient households (lenders) decrease their consumption. These effects peak in the period just prior to default. Impatient households cannot borrow in the default state, which causes a misallocation of housing where the patient households own all of the housing stock, lowering its marginal utility and price once default actually occurs.

This paper augments the New-Keynesian model with a housing market that includes both owner occupied housing and rental housing. In this model, as well as the related literature, the economy is populated by impatient households and patient households. The patient households are the lenders and the impatient households are the borrowers, identified by a lower discount factor and typically lower wealth. Housing acts as both a durable good and collateral on secured loans made by the lenders to borrowers. If housing prices are sufficiently low, borrowers lack sufficient

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<sup>1</sup>Taken from "Operation HOPE Global Financial Dignity Summit," Atlanta, Georgia, November 15, 2012.

<sup>2</sup>See, for example, Kiyotaki and Moore (1997), Iacoviello (2005) and Bernanke and Gertler (2001).

collateral to repay their loans and they all default.<sup>3</sup> In related settings, Bernanke and Gertler (2001) and Iacoviello (2005) examine New-Keynesian models with asset markets (stocks and housing respectively) and find evidence that credit constraints act to magnify and increase the persistence of demand shocks. However, these papers do not allow borrowers to take out loans greater than the discounted future asset value and consequently borrowers cannot default.<sup>4</sup>

We explicitly allow for default in order to analyze the effect of insolvency on the economy. In the case in which default is characterized by a temporary loss of access to housing and credit markets, there are three mechanisms through which default could potentially effect the economy. First, default represents a transfer of wealth from lenders to borrowers. This transfer occurs both because borrowers are unable to meet their loan obligations, and because the value of seized collateral (housing) declines as a result of default. Second, default creates a misallocation of housing, where lenders retain most or all of the economy's owner occupied housing stock. Third, by losing access to credit markets, borrowers lose the ability to finance consumption.

We consider (in Section 5) the effects of default within a single period. Here, the economy enters the period with a predetermined default probability. We then compare pairs of shocks that result in default with those that do not. We find:

1. Sufficiently positive demand shocks increase the marginal utility of consumption and induce households to substitute away from housing to consumption. This lowers housing prices and results in default.
2. Sufficiently negative productivity shocks reduce the level of consumption and increase consumption's marginal product. Households again substitute away from housing, lowering its price and inducing default.

We also find several large and discrete effects of default on the model's endogenous variables:

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<sup>3</sup>For tractability, the model assumes a single representative borrower and a single representative lender. As a result, either none or all borrowers default in each period.

<sup>4</sup>There is a partial equilibrium literature that allows agents to default but macroeconomic effects are not explored. See Gale and Hellwig (1985), for example.

3. Impatient household consumption increases. This is because their increased wealth and inability to buy housing dominate their inability to finance consumption through debt.
4. Patient household consumption declines due to their reduced wealth. This effect is also large enough to cause aggregate consumption and output to decrease.<sup>5</sup>
5. Housing prices fall. This is because housing is misallocated by being fully owned by the patient households. This reduces its marginal utility and price.

We also look at the case where there is no loss of access to housing or credit markets.<sup>6</sup> We show that the effects of default are small in this case, suggesting that the loss of access to financial markets is the crucial, and previously unmodeled, aspect of mortgage default.

We then (in Section 6) examine the behavior of the model over time. In contrast to Section 5, the probability of default varies over time, and it tends to rise before default actually occurs. We observe a common pattern where impatient households respond to higher default risk by increasing their consumption while patient households reduce theirs. The latter effect is larger so that aggregate consumption declines. As a result, changes in consumption tend to be most pronounced just prior to the default period. In contrast, the misallocation of housing that occurs during default causes decreased housing prices to be most pronounced during the default period itself, instead of before it.

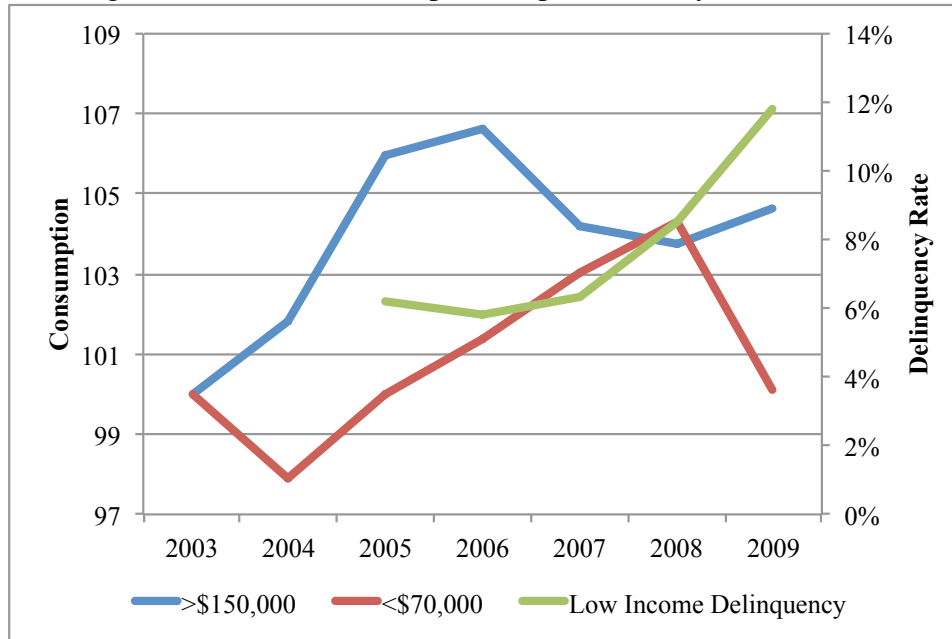
Our results are consistent with the U.S. data for the period around 2007 when mortgage delinquencies began to rise dramatically. Figure 1 plots average personal consumption expenditures using data from the Bureau of Labor Statistics for households with incomes over \$150,000 and for those with incomes under \$70,000 with both series normalized at 100 for their 2003 values. In our model, patient households are much wealthier than impatient households and the former time-series may thus represent lenders while the latter represents borrowers. Figure 1 also plots the mortgage delinquency rate.

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<sup>5</sup>Because the model does not include capital and the housing stock is fixed, aggregate output and consumption are the same.

<sup>6</sup>The scenario where households lose access to credit, but not housing markets, is similar to the case where agents lose access to both markets.

Figure 1: Personal Consumption Expenditures by Income Level



These data are, of course, influenced by many factors not included in our model. But it is noteworthy that for this one prominent example, the data behave similarly to our model. As the probability of default increases, poorer households consume more. In our model, poor households realize that foreclosure is likely and increase their consumption while they are still able. Wealthy households, however, anticipate a reduction in their wealth as default risk rises and respond by reducing their consumption.

Section 6 examines the effect of varying impatient households' access to credit, including cases where they are allowed to go underwater on their mortgage debt by making payments even when they owe more than the value of their collateral. As borrowers are able to go further underwater, they have higher debt payments, their consumption decreases, and default becomes less common. At the same time, patient households are able to consume more. However, the decrease in impatient households' steady state utility is greater than the increase in patient households' utility. If the policymaker places at least as much weight on the utility of the poor, it is optimal to place restrictions on how far underwater borrowers are allowed to be.

Section 6 also examines monetary policy by allowing the central bank to include asset prices in an interest rate rule. The main result is that increasing interest rates along with housing prices tends to dampen the policy response to inflation, making prices less stable. Because debt is not indexed to inflation, more volatile prices increase the frequency of default.

The paper is organized as follows. Section 2 discusses the related literature including empirical evidence that supports our model's main conclusions. Section 3 develops the model. Section 4 develops the adaptive learning algorithm that describes how expectations are formed.<sup>7</sup> Section 5 illustrates the causes and effects of mortgage default in a single period. Section 6 reports simulation results that depict model dynamics. Finally, Section 7 concludes.

## 2 Related Literature

Our paper contributes to the extensive literature, first developed by Kiyotaki and Moore (1997), on credit constraints. In this literature, an asset (housing, capital, land, etc.) acts both as an input to production and serves as collateral on secured loans by limiting borrowers' credit to an amount less than or equal to the value of their collateralized assets.<sup>8</sup> In this literature, credit constraints are a powerful transmission mechanism for the amplification and propagation of shocks. A shock that reduces the price of collateral also restricts access to credit, which reduces demand for the assets, further lowering its price, and the cyclical process continues. This financial accelerator effect helps to explain how relatively small shocks can result in large business cycle fluctuations.

Several papers extend the financial accelerator mechanism to a New-Keynesian framework. The closest paper to ours is Iacoviello (2005). He augments a New-Keynesian model with a housing sector where borrowers secure debt with their nominal housing wealth. Consistent with previous papers in the financial accelerator literature, the real economic effects of demand shocks are amplified and propagated due to credit market frictions. A negative demand shock drives down

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<sup>7</sup>See Evans and Honkapohja (2009) for a more detailed discussion of adaptive learning.

<sup>8</sup>Numerous empirical studies estimate that many households are credit constrained. See Zeldes (1989), Jappelli (1990), and Campbell and Mankiw (1989).

both goods and housing prices, which tightens the credit constraint, and allows agents to borrow less, further reducing aggregate demand. Quantitatively, a one-standard deviation increase in the interest rate decreases output by 3.33 % in the absence of credit limits, but by 3.82 % when the credit channel is included. However, Iacoviello (2005) finds that the result is reversed when the economy experiences an adverse supply shock. The collateral constraint acts a “decelerator” of supply shocks in that an adverse supply shock increases housing prices and thus has a positive effect on borrowers net worth.<sup>9</sup>

Each of these papers abstracts away from actual default. The potential for strategic default motivates the credit constraint, but default never actually occurs. Our paper allows for mortgage default in the New Keynesian framework. In doing so, we show that default provides an additional source of amplification that has not previously been modeled.

Our paper is also related to a separate literature that does model actual default. Gale and Hellwig (1985) solve for the optimal debt contract in a framework where asymmetric information motivates costly verification in which the state is only observed if the firm is insolvent. In this setting, the optimal credit contract is the standard debt contract with bankruptcy. If the firm is insolvent, the lender can repossess as much of the firm’s debt as possible in the form of assets; there is not, however, a credit constraint on the borrower. Thus, the lender recovers what they can of the loan, but there is no guarantee it will be equal to the full amount of the debt. We adopt this approach and apply it to our business cycle model with housing.

Other papers use this structure in very different general equilibrium models. Fiore and Tristani (2013), and Goodhart et al. (2009) model commercial default. Faia (2007) uses it to model default between countries in an open economy DSGE model. Our focus, however, is on mortgage default. To the best of our knowledge, ours is the first paper to model mortgage default in a New Keynesian setting.

Our paper examines the performance of monetary policy that relies on a Taylor type rule. We are specifically interested in how a response to housing prices affects macroeconomic performance.

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<sup>9</sup>Other papers in this area include Bernanke et al. (1999). In this paper, stocks instead of housing act as collateral.

We find that increasing interest rates as real housing prices increases the frequency of default. Such a policy benefits borrowers and harms lenders. Furthermore, such a policy is desirable for any non-convex social welfare function. This result is in contrast to the related literature, including Iacoviello (2005) and Bernanke and Gertler (2001), which finds negligible benefits to including asset prices in the monetary policy rule. Those papers, however, do not include actual default which we find has substantial real effects on the model.

Because we rely on adaptive learning to solve our general equilibrium model, our paper also contributes to the literature on learning and monetary policy. Orphanides and Williams (2008) compare optimal policy under learning and rational expectations and find that learning provides an additional incentive to manage inflationary expectations. Therefore, it is optimal for policymakers to more aggressively respond to inflation under learning. Xiao (2013) examines optimal policy in a New-Keynesian model with housing. He finds that the optimal response to housing prices is sensitive to the specific information set of agents. Evans and McGough (2005) examine learning in a standard New-Keynesian model. They find that the condition for determinacy of equilibrium is usually, but not always, the same as the condition for stability under learning.

## 2.1 Empirical Evidence

Ample empirical evidence supports our findings that default directly contributes to economic downturns. Several microeconomic studies investigate the relationship between housing prices and foreclosures and the general consensus is that there is a negative relationship between foreclosures and housing prices.<sup>10</sup> There is also a well-documented relationship between price and foreclosures in the reverse direction (a decrease in housing prices causes foreclosures).<sup>11</sup>

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<sup>10</sup>Gerardi et al. (2012) discover that the magnitude of this effect actually peaks before the foreclosure process is complete. We find a similar result, that the impact of default is most intense in the period prior to foreclosure. See Section 6 for more details. Lin et al. (2009) observe that the negative relationship between housing prices and foreclosures is larger during recessions. Leonard and Murdoch (2009) study the real estate market in Dallas and find that the impact of foreclosures on neighboring house prices is decreasing in distance from the foreclosed home. Harding et al. (2009) obtain a similar result using data from 37 MSAs in 13 states. Rogers and Winter (2009) use St. Louis county data and conclude that there is a non-linear effect of foreclosures on real estate prices. The marginal impact of an additional foreclosure decreases as the number of foreclosures increases.

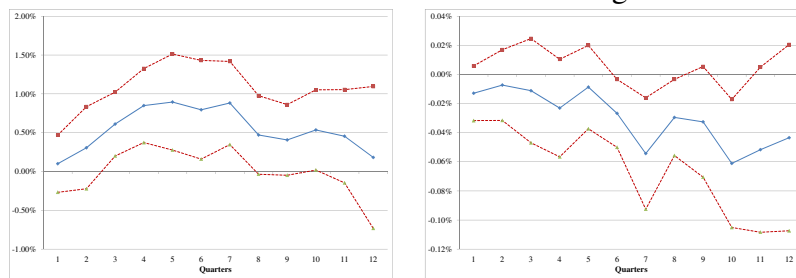
<sup>11</sup>See Foster and Order (1984), Gerardi et al. (2008), Bajari et al. (2008), and Rana and Shea (2014).



The direction of causality in the relationship between foreclosures and housing prices is unclear; in fact, most studies find evidence of dual causality. Foreclosure will occur if and only if the homeowner is underwater, inherently linking housing prices and foreclosures regardless of whether foreclosures have a direct impact on real estate prices. Mian et al. (2014) use a two-staged least squares analysis to correct for endogeneity employing whether states have judicial foreclosure laws or non-judicial foreclosure laws as an instrument. In addition, Mian et al. (2014) examine the real effects of foreclosures and housing price on real economic activity. Mian et al. (2014) find that from 2007 to 2009, foreclosures caused a 20% to 30% decline in housing prices, 15% to 25% decline in residential investment, and a 20% to 35% decline in auto sales. Calomiris et al. (2013) use state level data in a panel vector autoregression (PVAR). They find that the effect housing prices have on foreclosures is larger than the effect foreclosures have on housing prices (quantitatively, 79% greater).

A small macroeconomic literature examines how shocks to the housing sector affect the general economy. Rana and Shea (2014) uses a local projection method to estimate a system that includes foreclosures, unemployment, and housing prices. They find that shocks to foreclosures have very large effects on unemployment: an impulse that increases foreclosures by 0.118% after nine quarters causes unemployment to rise by 6.4% after three years. Shocks to housing prices have much smaller, and often insignificant effects. The impulse response functions for each shock on unemployment are reproduced below:

Figure 2: Response of Unemployment to Shocks to Foreclosures and Housing Prices



These empirical results are consistent with the mechanism we model in this paper, implying

that foreclosures have real effects on macroeconomic variables and directly contribute to business cycle fluctuations. We now build a formal model that captures the relationship between foreclosures, housing prices, and real economic activity that is found in the empirical literature.

### 3 Model

We develop a discrete time, infinite horizon model, populated by impatient and patient households. Following Iacoviello (2005), we assume that a set of patient households have relatively high discount factors,  $\gamma$ , and thus typically lend to a separate set of impatient households who have lower discount factors,  $\beta < \gamma$ .<sup>12</sup> Our model has four notable differences from Iacoviello (2005). First, we allow for actual mortgage default instead of simply assuming that the threat of default (which never occurs in Iacoviello (2005)) imposes a credit constraint on borrowers.<sup>13</sup> Second, we add a rental housing market. This addition allows us to better examine the effects of impatient households potentially losing access to owner occupied housing and instead being forced into the rental market exclusively. Third, to solve the model (which contains several discontinuities) we rely on adaptive learning in the non-linear model instead of linearizing the model and using rational expectations. Finally, for simplicity, we assume that housing does not act as an input in the production function.

Households work, consume, supply and demand rental housing, and demand real estate.<sup>14</sup> Throughout this section, we denote variables that correspond to patient households with a prime symbol. We begin by assuming the following utility function for patient households:

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<sup>12</sup>Rarely, impatient households lend to patient households in equilibrium. In this case, the model is unchanged except that default risk applies to the patient households. However, to remain consistent with the related literature, we use borrowers and impatient households interchangeably hereafter.

<sup>13</sup>Note, we assume that all cases of default result in foreclosure. In the data, these terms are not synonymous; however, in our model, we use them interchangeably.

<sup>14</sup>Note, money is not explicitly included in the utility function. Instead, we assume that the monetary authority can directly set the nominal interest rate. It would be trivial to add money in the utility function and derive the corresponding money demand equation. Household utility in Iacoviello (2005) depends on money balances; however, he only examines interest rate rules for which money supply will always equal money demanded in equilibrium. Given these conditions, the quantity of money does not affect the rest of the model and is disregarded.

$$u(c'_t, l'_t, h'_t, x'_t, x_t) = e_t \ln(c'_t) + j \ln\left(\left[(h'_t - x_t)^\epsilon + \omega(x'_t)^\epsilon\right]^{\frac{1}{\epsilon}}\right) - \frac{(l'_t)^2}{2} \quad (3.1)$$

where  $e_t$  is an exogenous demand shock. We assume that patient households may rent housing from impatient households ( $x'_t$ ) at the rental rate ( $v'_t$ ). For computational reasons, we do not allow patient households to rent housing to each other.<sup>15</sup> The variable  $h'_t$  represents patient households' homeownership, which implies that  $(h'_t - x_t)$  is their level of owner occupied housing. The parameter  $\epsilon$  captures the degree of substitutability between owner occupied and rental housing, and the term  $\omega \leq 1$  allows us to assume that households inherently prefer the former. Assuming for the moment that no default occurs, the budget constraint is described by

$$\frac{A_t (l'_t)^\alpha}{m_t} - b_t + k'_t + v_t x_t = c'_t + q_t (h'_t - h'_{t-1}) + v'_t x'_t - R_{t-1}^m b_{t-1} / \pi_t + R_{t-1} k'_{t-1} / \pi_t + F_t \quad (3.2)$$

The exogenous variable  $A_t$  is a random, AR(1), productivity shock. The variable  $q_t$  represents the price of housing. We assume that patient households may borrow from each other ( $k'_t$ ) at the riskless rate  $R_t - 1$ . As is standard, in equilibrium,  $k'_t = 0$ . The variable  $b_t$  represents impatient household debt to patient households, and the variable  $R_t^m - 1$  represents the corresponding risky interest rate. By including  $\pi_t$  in (3.2), we are assuming that debt is not indexed to inflation.<sup>16</sup>

Finally, we assume that households produce intermediate goods, which are then sold to a retail sector which costlessly transformed into final goods by a retail sector. The price of final goods is marked up at rate,  $m_t$ . We assume that the patient households own the retailers and receives profits  $F_t = (1 - m_t^{-1}) A_t [l_t^\sigma + l'_t{}^\sigma]^{\frac{1}{\sigma}}$ .

Optimization yields the following first-order conditions:

$$\frac{\alpha A_t e_t (l'_t)^{\alpha-1}}{c'_t m_t} = l'_t \quad (3.3)$$

<sup>15</sup>We also assume that impatient households may not rent to each other, which results in two separate rental rates.

<sup>16</sup>As discussed in Iacoviello (2005), most U.S. debt is not indexed to inflation.

$$\frac{j\omega(x'_t)^{\epsilon-1}}{v'_t [(h'_t - x_t)^\epsilon + \omega(x'_t)^\epsilon]} = \frac{e_t}{c'_t} \quad (3.4)$$

$$\frac{e_t}{c'_t} = \gamma E_t \left[ e_{t+1} \frac{R_t}{c'_{t+1} \pi_{t+1}} \right] \quad (3.5)$$

$$\frac{j(h'_t - x_t)^{\epsilon-1}}{q_t [(h'_t - x_t)^\epsilon + \omega(x'_t)^\epsilon]} + \gamma E_t \left[ \frac{e_{t+1} q_{t+1}}{q_t c'_{t+1} \pi_{t+1}} \right] = \frac{e_t}{c'_t} \quad (3.6)$$

$$\frac{e_t v_t}{c'_t} = \frac{j(h'_t - x_t)^{\epsilon-1}}{[(h'_t - x_t)^\epsilon + \omega(x'_t)^\epsilon]} \quad (3.7)$$

Equation (3.3) is the labor supply rule. Equation (3.4) is the rental demand equation. Equation (3.5) is a standard consumption Euler equation. The housing demand equation is equation (3.6). Equation (3.7) is the rental supply equation and simply equates the consumption that results from renting out an additional unit of housing to the utility of using that housing as owner occupied housing.

In addition, the patient household must choose to distribute its lending between other patient households ( $k_t$ ) and impatient households ( $b_t$ ). The patient household does so taking the risky and risk free interest rates, and the probability of default,  $p(def)$ , as given. Optimization then yields an interest rate arbitrage condition:

$$E_t \left[ \frac{e_{t+1} R_t}{c'_t} \right] = (1 - p(def)) E_t^* \left[ \frac{e_{t+1} R_t^m}{c'_{t+1}} \right] + p(def) E_t^{**} \left[ \frac{e_{t+1} R_t^m rec_{t+1}}{c'_{t+1}} \right] \quad (3.8)$$

where \* indicates the conditional expectation in the case of no default and \*\* indicates the conditional expectation in the case of default. The recovery rate  $rec_t$  is the perceived rate of recovery in the case of default.

We now consider the impatient households. Their optimization problem is similar to that of the patient households, except that they potentially default on their mortgage debt and thus borrow from the patient households at a risky interest rate. In addition to choosing consumption, various

types of housing, debt, and labor, they also implicitly choose their probability of default.

$$u(c_t, l_t, h_t, x_t, x'_t) = e_t \ln(c_t) + j \ln\left(\left[(h_t - x'_t)^\epsilon + \omega x_t^\epsilon\right]^{\frac{1}{\epsilon}}\right) - \frac{l_t^2}{2} \quad (3.9)$$

We assume that the demand shock affects both types of households identically. Again, ignoring the potential for default, the impatient household's budget constraint is:

$$\frac{A_t l_t^\alpha}{m_t} + b_t = c_t + q_t(h_t - h_{t-1}) + v_t x_t - v'_t x'_t + R_{t-1}^m b_{t-1} / \pi_t \quad (3.10)$$

Optimization yields:

$$\frac{j \omega x_t^{\epsilon-1}}{v_t [(h_t - x'_t)^\epsilon + \omega x_t^\epsilon]} = \frac{e_t}{c_t} \quad (3.11)$$

$$\frac{\alpha A_t e_t l_t^{\alpha-1}}{c_t m_t} = l_t \quad (3.12)$$

$$\frac{e_t}{c_t} = \beta E_t^* \left[ \frac{e_{t+1} R_t^m}{c_{t+1} \pi_{t+1}} (1 - p(def)) \right] + \beta E_t^* \left[ \frac{\partial p(def)}{\partial b_t} \right] \Gamma_t \quad (3.13)$$

$$\frac{j(h_t - x'_t)^{\epsilon-1}}{q_t [(h_t - x'_t)^\epsilon + \omega x_t^\epsilon]} + \beta E_t^* \left[ \frac{e_{t+1} q_{t+1}}{q_t c_{t+1} \pi_{t+1}} (1 - p(def)) \right] = \frac{e_t}{c_t} + \frac{\beta}{q_t} E_t^* \left[ \frac{\partial p(def)}{\partial h_t} \right] \Gamma_t \quad (3.14)$$

$$\frac{e_t v'_t}{c_t} = \frac{j(h_t - x'_t)^{\epsilon-1}}{[(h_t - x'_t)^\epsilon + \omega x_t^\epsilon]} \quad (3.15)$$

Equation (3.11) is the rental demand equation, and (3.12) is the labor supply equation. Equation (3.13) is the Euler equation. Impatient households must consider both the probability of default as well as the utility loss resulting from a potential loss of access to markets, denoted  $\Gamma_t$ . Increased debt ( $b_t$ ) increases the probability of a welfare reducing loss of access to financial markets following default. Due to this default risk, impatient households borrow at the risky rate  $R_t^m - 1$  instead

of the riskless rate  $R_t - 1$ .

Equation (3.14) is the housing demand equation. Once again, impatient households must factor in how their choice of housing affects default risk. Higher values of  $h_t$  increase collateral and make default less likely. The rental supply equation is (3.15).

We fix the housing stock equal to one:

$$h_t + h'_t = 1 \quad (3.16)$$

and we assume a standard Calvo mechanism that allows a fixed fraction,  $\theta$ , of intermediate goods producers to re-optimize their price. Their price setting problem then yields:

$$m_t = E_t \left[ \frac{\pi_t^{\frac{\gamma}{\lambda}} \pi_t^{\frac{-1}{\lambda}}}{\sigma} \right] \quad (3.17)$$

where  $\lambda = \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta(1-\alpha(1-\frac{1}{1-\sigma}))}$ .<sup>17</sup>

Finally, the monetary authority sets the nominal interest rate using the following non-linear policy rule:

$$R_t = \bar{\pi} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left( \frac{q_t}{\bar{q}} \right)^{\phi_q} \quad (3.18)$$

where the inclusion of  $\phi_\pi$ , and  $\phi_q$  allow the monetary authority to potentially respond to both inflation and asset prices.

The model includes the following 16 endogenous variables:  $c_t, c'_t, x_t, x'_t, h_t, h'_t, l_t, l'_t, b_t, v_t, v'_t, q_t, R_t, R_t^m, m_t, \pi_t$ . It consists of eleven first-order conditions, one budget constraint, (3.16) through (3.18) and a combined production function:

$$c_t + c'_t = A_t \left[ l_t^\sigma + (l'_t)^\sigma \right]^{\frac{1}{\sigma}} \quad (3.19)$$

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<sup>17</sup>See Woodford (2003) for details of this equation's derivation.

### 3.1 Default

We assume that credit markets work as follows. At the start of each period, the credit market clears, requiring that borrowers sell off assets in order to pay off their debt. Borrowers do not choose whether to default. If they have sufficient capital, then the debt is fully repaid. If they do not, then the model enters the default state. Borrowers have sufficient collateral if and only if  $q_t h_{t-1} \pi_t \geq R_{t-1}^m b_{t-1}$ .<sup>18</sup> If borrowers are able to pay their debt, all agents then make their labor supply, consumption, and savings choices.

If  $q_t h_{t-1} \pi_t < R_{t-1}^m b_{t-1}$ , then default occurs. Lenders seize all of the borrowers' assets, and we then consider several cases where borrowers lose access to financial markets for the period when default occurs:

1. *No access to housing or bond markets.*

In this case, impatient households may not purchase housing or access credit markets. Here,  $b_t = h_t = x'_t = 0$ , and  $R_t^m$  and  $v'_t$  are undefined. Equations (3.2), (3.4), (3.13) through (3.15), and (3.8) no longer describe the model's equilibrium. The following condition, however, must hold:

$$\frac{A_t l_t}{m_t} = c_t + v_t x_t \quad (3.20)$$

Equation (3.20) is simply the budget constraint for impatient households when they lack access to housing and credit.

2. *No access to bond markets.*

Here, impatient households may purchase housing, but may not borrow using the credit market.<sup>19</sup> In this case,  $b_t = 0$ , (3.2) and (3.15) no longer bind, but the following budget constraint holds in equilibrium.

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<sup>18</sup>In Section 6, we relax this condition and allow households to go underwater on their debt where they owe more than the value of their collateral.

<sup>19</sup>We do not consider the case where impatient households may access bond, but not housing markets, because such a scenario ensures that default will occur in the next period.

$$\frac{A_t l_t}{m_t} + v_t' x_t' = c_t + v_t x_t + q_t h_t \quad (3.21)$$

### 3. *Writedown.*

Here, impatient households may continue to access both housing markets and credit markets. We further assume that unpaid debt is written off, equivalent to imposing  $h_{t-1} = b_{t-1} = 0$ , and that the model is otherwise unchanged. This case (and case 4 where no default occurs), are not true equilibria because the utility loss from losing access to housing and credit markets,  $\Gamma$ , is equal to zero. For such a value, impatient households would wish to borrow an infinite amount and the model is not well defined.<sup>20</sup>

In calculating this case (and the no default case), we impose the value of  $\Gamma$  from Case 1. This scenario thus represents a one-time, unexpected deviation from that case where borrowers, for some reason, maintain access to credit markets. We present it because, by comparing it to Case 1, we are able to quantify the consequences of losing access to financial markets in the default period.

### 4. *No default.*

In this scenario, we ignore the default condition and continue the model unaffected.

## 4 Adaptive Learning and Expectations Formation

The most common approach for modeling expectations is rational expectations. Under rational expectations, agents are assumed to use the model's reduced form solution to form mathematically optimal forecasts. Using rational expectations has been criticized for requiring agents to possess implausibly high amounts of information. For example, they must know the exact model generating the data, despite the field's disagreement over which model is best and an innumerable list of

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<sup>20</sup>If agents know that debt is simply written down, then  $p(def) \rightarrow 1$  and  $R_t^m \rightarrow \infty$  and the model's equilibrium is no longer well defined.



candidates. In addition, they must also know the model's correct calibration and the true nature of its stochastic shocks.

The most prominent alternative to rational expectations is adaptive learning.<sup>21</sup> Adaptive learning is motivated by the principle of cognitive consistency, which suggests that agents in the model be neither much less intelligent nor much smarter than the people modeling them. Thus, adaptive learning assumes that agents use econometric algorithms (ordinary least squares, in this paper) to form expectations. Perhaps the most compelling argument for learning is that the reader, if asked to form forecasts of variables such as consumption and housing prices, is most likely to rely on econometrics rather than simply conjuring up a rational expectation based on minimal data (lagged productivity, housing prices and debt, along with current shocks, and whether the economy is in default). He would behave like an adaptive learner. Learning also provides an additional benefit, allowing us to work with the non-linear equations as opposed to taking linear approximations.

We assume a simple type of learning where agents fit most variables to one-lag autoregressive, AR(1), processes.<sup>22</sup> Furthermore, to simplify the analysis, we assume that agents do not consider whether or not the economy is in the default state when fitting the model. For  $w_t = c_t, c'_t, \pi_t, q_t$ , we assume that agents form expectations using:

$$w_t = a_w + b_w(w_{t-1} - a_w) + u_t \quad (4.1)$$

where  $a_w$  and  $b_w$  are regression coefficients obtained through recursive least squares:

$$\begin{bmatrix} a'_{w_t} \\ b_{w_t} \end{bmatrix} = \begin{bmatrix} a'_{w_{t-1}} \\ b_{w_{t-1}} \end{bmatrix} + t^{-1} R_t^{-1} \begin{bmatrix} 1 \\ w_{t-1} \end{bmatrix} (w_t - a'_{w_{t-1}} - b_{w_{t-1}} w_{t-1}) \quad (4.2)$$

$$R_t = R_{t-1} + t^{-1} \left( \begin{bmatrix} 1 \\ w_{t-1} \end{bmatrix}^2 - R_{t-1} \right) \quad (4.3)$$

<sup>21</sup>For a detailed treatment of adaptive learning, see Evans and Honkapohja (2001).

<sup>22</sup>Experimental evidence suggests that agents do use simple autoregressive processes to form expectations. See, for example, Hommes et al. (2005).

$$a_w = \frac{a'_w}{1 - b_w} \quad (4.4)$$

It then follows that agents use (4.1) to form expectations according to:

$$E_t[w_{t+1}] = a_w + b_w(w_t - a_w) \quad (4.5)$$

Agents also use this algorithm to estimate the default distribution. We assume that rely on point expectations so that:

$$E_t[q_{t+1}\pi_{t+1}] = E_t[q_{t+1}]E_t[\pi_{t+1}] \quad (4.6)$$

Agents obtain an estimate of the expectational error from (4.6) using the following process:

$$\sigma_{q\pi,t} = \sigma_{q\pi,t-1} + t^{-1} |(E_{t-1}[q_t]E_{t-1}[\pi_t] - q_t\pi_t)| \quad (4.7)$$

Agents then fit (4.7) to a truncated normal distribution so that:

$$E_t \left[ \frac{\partial p(def)}{\partial b_t} \right] = \frac{R_t^m}{h_t} g \left( \frac{E_t[q_{t+1}\pi_{t+1}] - \frac{b_t R_t^m}{h_t}}{\sigma_{q\pi}} \right) \quad (4.8)$$

$$E_t \left[ \frac{\partial p(def)}{\partial h_t} \right] = -\frac{b_t R_t^m}{h_t^2} g \left( \frac{E_t[q_{t+1}\pi_{t+1}] - \frac{b_t R_t^m}{h_t}}{\sigma_{q\pi}} \right) \quad (4.9)$$

where  $\left( \frac{E_t[q_{t+1}\pi_{t+1}] - \frac{b_t R_t^m}{h_t}}{\sigma_{q\pi}} \right)$  is the truncated normal probability density function.<sup>23</sup>

Agents must obtain an estimate for  $\Gamma_t$ , the utility loss from losing access to housing and credit markets. We assume they do so by comparing equilibrium in the default state with the hypothetical equilibrium had they been allowed access to the relevant credit markets. The variable  $\Gamma$  is only updated in the default state:

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<sup>23</sup>We truncate the normal distribution at two standard deviations. For values below -2 standard deviations, the distribution is then linear until  $b_t = 0$  where  $g(\cdot) = 0$ . For values above 2 standard deviations, it is linear until 12 standard deviations where  $g(\cdot) = 0$ .

$$\Gamma_t = (1-(t^*)^{-1})\Gamma_{t-1} + (t^*)^{-1}E_t \left[ u(\hat{c}_t, \hat{h}_t, \hat{x}_t, \hat{l}_t) - u(c_t, h_t, x_t, l_t) + \frac{q_{t+1}\hat{h}_t - \hat{R}_t^m \hat{b}_t}{c_{t+1}} \right] \quad (4.10)$$

where “hats” indicate the values of variables in a version of the model where impatient households maintain access to all markets and  $t^*$  is the sample size of default periods. Individual households solve this problem taking all prices as given.

Finally, agents also update their estimate of the recovery rate in the default state only. This is obtained by:

$$rec_t = rec_{t-1} + (t^*)^{-1} \left( \frac{q_t \pi_t h_{t-1}}{b_{t-1} R_{t-1}^m} - rec_{t-1} \right) \quad (4.11)$$

Table 1 reports our calibration of exogenous parameters. Most of our values are taken from Iacoviello (2005). We set  $\omega = 0.9$ , so that there is a slight inherent preference for owner occupied housing and  $\epsilon = 0.6$ , implying an intermediate degree of substitutability between owner occupied and rental housing.

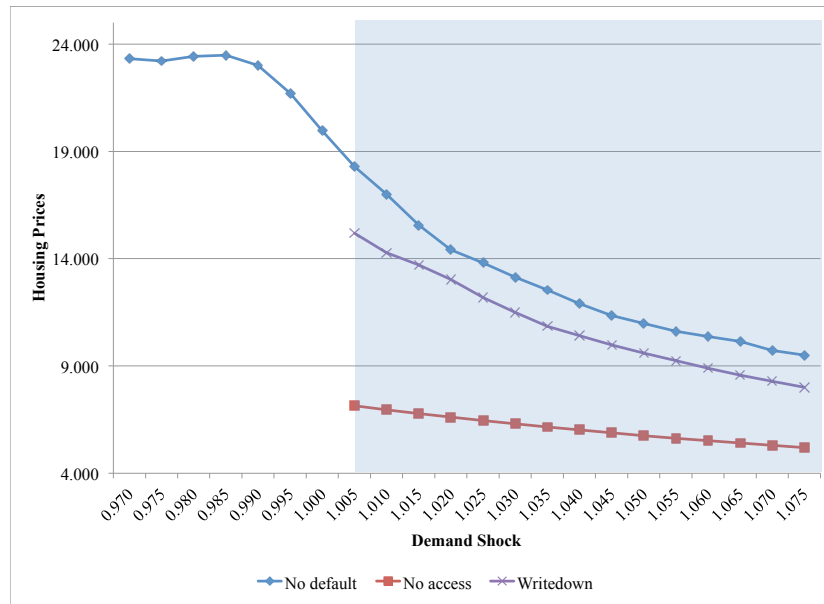
$j$	weight on housing	0.10
$\epsilon$	substitutability of housing types	0.60
$\alpha$	labor's share in production function	0.67
$\beta$	impatient households' discount factor	0.90
$\gamma$	patient households' discount factor	0.99
$\omega$	weight on rental housing	0.90
$\phi_\pi$	policy response to inflation	2.00
$\sigma$	substitutability of consumer goods	0.71
$\theta$	degree of price stickiness	0.67
$\sigma_a$	st. dev. of innovations to productivity	0.01
$\sigma_e$	st. dev. of innovations to demand	0.02
$\rho_a$	AR(1) coefficient for productivity shocks	0.95
$\rho_e$	AR(1) coefficient for demand shocks	0.00

## 5 Causes and Effects of Default

To examine what causes default and how default affects equilibrium, we consider the model at a fixed point in time for  $\phi_q = 0$ . We impose the initial conditions,  $h_{t-1} = 0.15$ , and  $b_t = 0.81$ , chosen so that the mean value of shocks is close to the default cutoff. The results that follow are representative of the model's systematic behavior. We begin by simulating this scenario for different values of the demand shock,  $e_t$ , holding  $A_t$  constant at 1.04. Figures 3 through 7 show the responses of different variables to various values of a demand shock (on the horizontal axis). As the value of the demand shock increases, the marginal utility of consumption also increases. Households thus substitute away from housing toward consumption. For large enough values of  $e_t$ , housing prices are low enough for the model to enter the default state, indicated by the shaded region. We report levels of the endogenous variables for under three of the four scenarios outlined in Section 3.1. We exclude the case where households lose access to bond, but not housing markets, because they are nearly identical to the case where borrowers lose access to both credit and housing markets.

Figure 3 displays the results for housing prices.

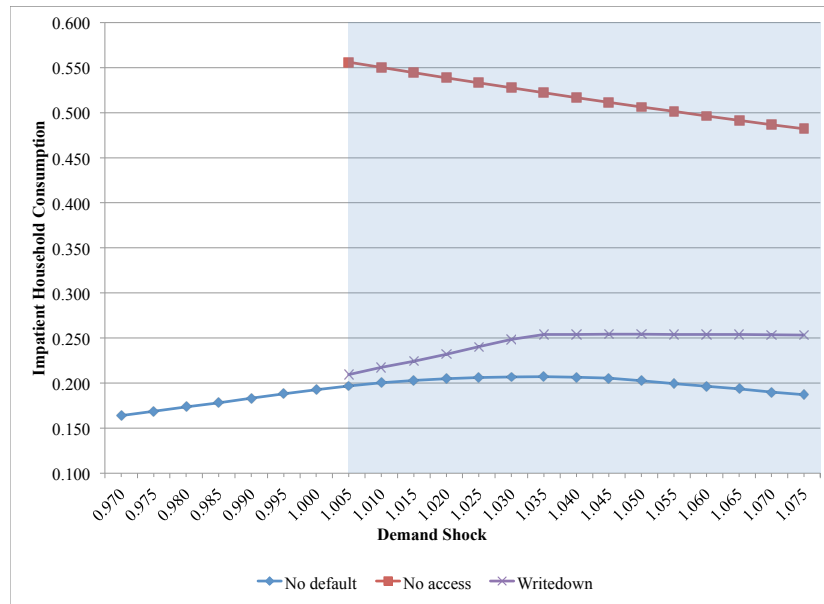
Figure 3: Behavior of  $q_t$



In this simulation, default occurs when  $e_t \geq 1.005$ . The top line ignores the default, and the middle line shows the case where debt is written off. The bottom line shows the case where impatient households lose access to housing and credit markets. This causes a discrete amplification of the decline in  $q_t$  that caused default to occur. The further reduction of housing prices occurs because default causes a misallocation of housing. Patient households own all housing when default occurs, resulting in a decreased marginal utility of housing, and lower housing prices.

Figure 4 illustrates the effects on impatient households' consumption.

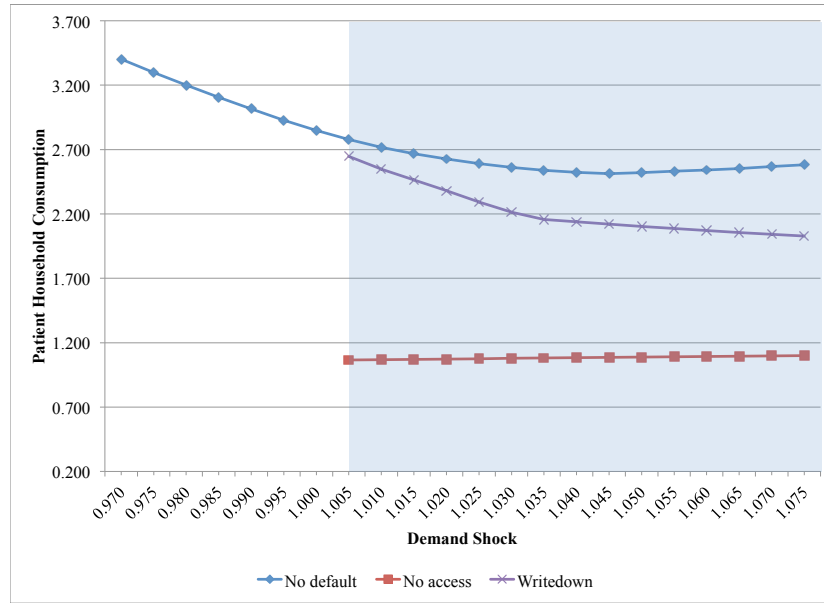
Figure 4: Behavior of  $c_t$



There are competing effects from default. The lack of access to credit markets reduces the ability of impatient households to borrow to finance consumption. Default also transfers wealth from patient to impatient households which increases consumption. The loss of access to housing also provides an incentive to substitute toward consumption. The latter effects dominate and  $c_t$  increases when default occurs.

Figure 5 illustrates the effects of default on patient households' consumption.

Figure 5: Behavior of  $c_t'$



Default transfers wealth from patient households to impatient households. Because impatient households recover housing as collateral when default occurs, the severe decline in housing prices shown in Figure 3 amplifies the scope of this wealth effect. As a result, default causes a discrete decrease in patient households’ consumption.

Comparing Figures 4 and 5, default causes a decrease in aggregate output (which equals aggregate consumption). Our theoretical model thus matches the empirical findings from Section 2.1—default is not simply a result of lower housing prices, it instead causes significant discrete decreases to both aggregate output and housing prices.

Figures 6 and 7 show the effects on housing that is rented from patient households to impatient households.

Figure 6: Behavior of  $x_t$

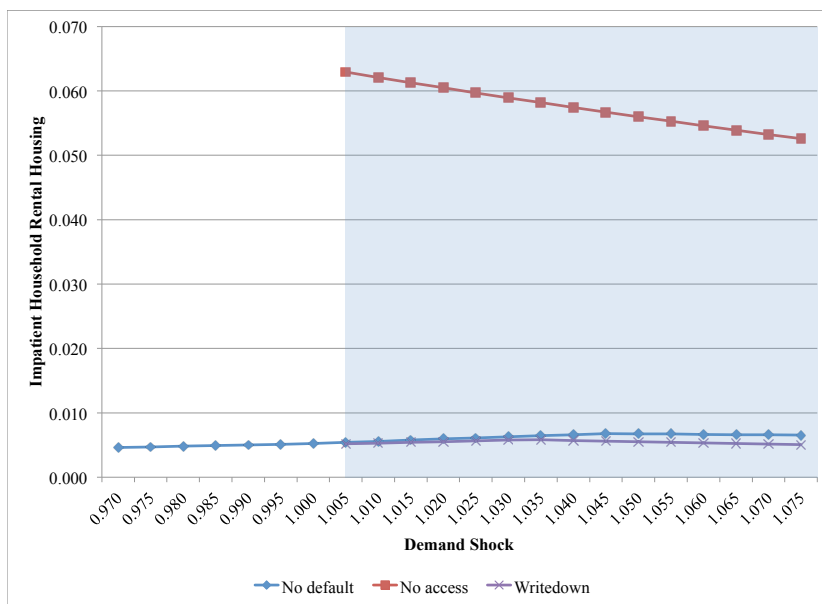
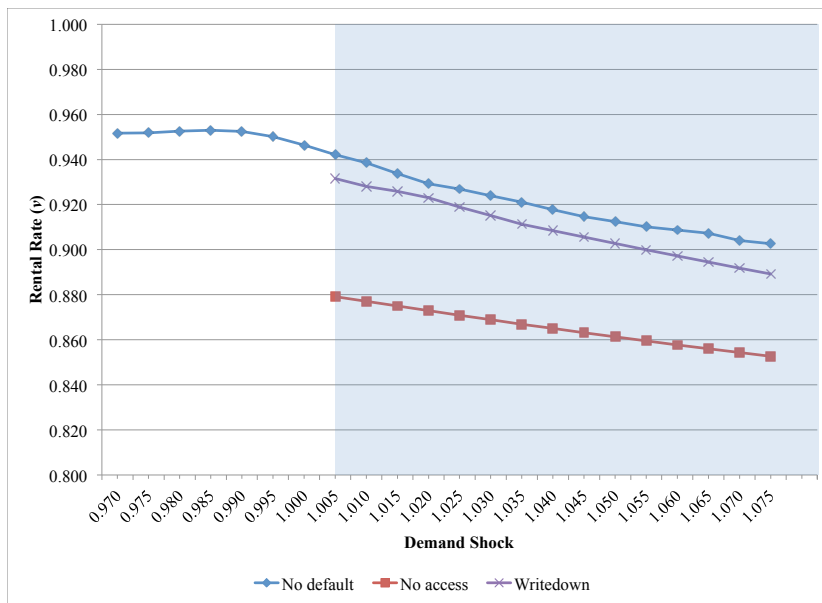


Figure 7: Behavior of  $v_t$



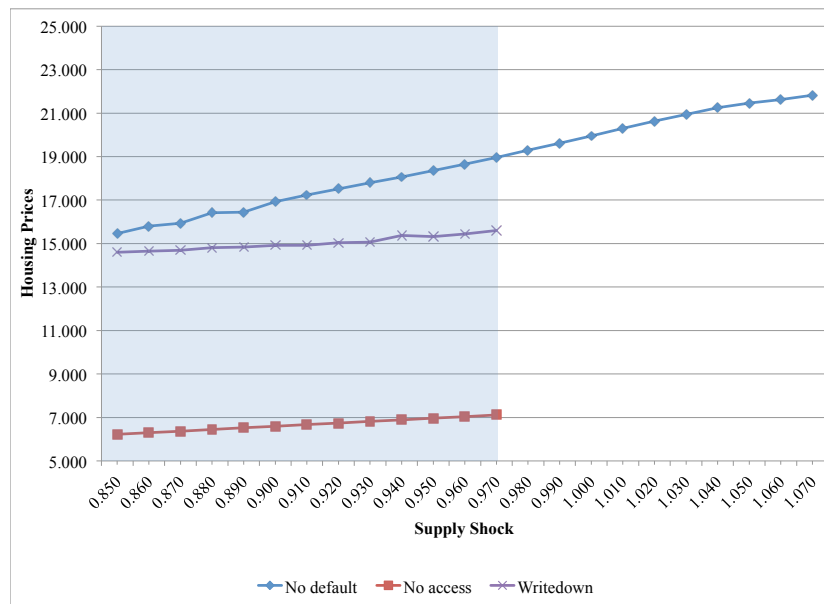


When impatient households lose access to owner occupied housing, they are forced into the rental housing market. As a result,  $x_t$  increases dramatically when default occurs. The effect on the rental rate is theoretically ambiguous because there is both increased supply and demand. In this simulation, the rental rate decreases, further amplifying the adverse wealth effect that patient households experience.

Adverse productivity shocks may also induce default in the model. As the supply shock,  $A_t$ , falls, so does output. The marginal utility of consumption decreases as the value of  $A_t$  increases. In this simulation default occurs if  $A_t \leq 1.03$ , holding  $e_t$  constant at 1.

Figure 8 plots housing prices under each scenario:

Figure 8: Behavior of  $q_t$  in Response to Productivity Shocks



As before, default creates a misallocation of housing that results in an amplification of the decline in housing prices. As the marginal utility of consumption increases, households wish to substitute away from housing toward consumption which reduces both housing prices and rents. The remaining effects of default, including a decline in aggregate consumption, are similar to the case where default results from a demand shock.

## 6 Simulation Results

We now examine how some alternate parameterizations affect the model. We begin by altering the default condition so that default occurs if and only if  $q_t h_{t-1} \pi_t < \chi R_{t-1}^m b_{t-1}$ . The parameter  $\chi$  represents how able households are to go underwater on their mortgage debt.  $\chi = 1$  implies that they cannot go underwater while higher values suggest that they are able to carry more debt. In much of the literature that uses credit constraints,  $\chi$  is interpreted as the share of collateral that is recoverable in the case of default, and is therefore constrained to be no greater than one. Empirical evidence, however, suggests that default generally occurs for higher values of  $\chi$ , between 1 and 1.5.<sup>24</sup> We thus examine the model in this range.

All simulations are for 5000 periods where the first 2000 are a burn for the learning process to converge. Learning coefficients are reported in Table 6.

Table 2: Equilibrium Dynamics for Different Values of  $\chi$ .

	$\chi = 1$	$\chi = 1.1$	$\chi = 1.2$	$\chi = 1.3$	$\chi = 1.4$	$\chi = 1.5$
Mean( $c_t$ )	0.645	0.643	0.631	0.623	0.594	0.527
St. Dev ( $c_t$ )	0.102	0.108	0.115	0.122	0.118	0.112
Mean( $c'_t$ )	1.052	1.080	1.115	1.097	1.178	1.302
St. Dev ( $c'_t$ )	0.156	0.176	0.191	0.201	0.223	0.218
Mean( $h_t$ )	0.253	0.242	0.233	0.252	0.203	0.166
St. Dev ( $h_t$ )	0.202	0.191	0.189	0.193	0.148	0.110
Mean( $q_t$ )	8.437	8.347	8.516	9.088	8.669	10.794
St. Dev ( $q_t$ )	1.198	1.204	1.313	1.366	1.559	1.951
Mean( $x_t$ )	0.019	0.019	0.019	0.015	0.013	0.013
St. Dev ( $x_t$ )	0.020	0.019	0.020	0.014	0.011	0.011
Mean( $x'_t$ )	0.084	0.078	0.074	0.085	0.058	0.050
St. Dev ( $x'_t$ )	0.084	0.076	0.070	0.079	0.055	0.034
Mean( $u_t$ )	-0.836	-0.856	-0.899	-0.914	-1.016	-1.239
Mean( $u'_t$ )	-0.050	-0.025	0.011	-0.008	0.077	0.206
p(def)	10.6%	8.6%	9.2%	3.0%	2.8%	3.1%

As  $\chi$  increases, impatient households are able to borrow with less collateral. Doing so results in higher interest payments that cause their consumption to fall while patient households' consumption increases. With an increased marginal utility of consumption and less need for housing

<sup>24</sup>See Fuster and Willen (2013).

as collateral, impatient households' share of the housing stock generally declines. Higher values of  $\chi$  cause competing effects to household utility, it increases for patient households while decreasing for impatient households. Simply adding their average utilities yields a maximum when  $\chi = 1.1$  with a similar value to where  $\chi = 1.0$ . Because impatient household utility is declining in  $\chi$ , any value of  $\chi > 1.1$  performs worse for any social welfare function that is non-increasing in utility inequality. Therefore, as long as the policymaker cares at least as much about impatient household utility as patient household utility, limitations should be placed on how far underwater impatient households can be. We would expect that as  $\chi$  increases, default would become less frequent. The results do illustrate a sharp fall in the frequency of the default state: for values of  $\chi$  less than or equal to 1.2, default occurs about 10.0% of the time. Higher values, however, yield default rates nearer to 3%.

We now compare, for the baseline case where  $\chi = 1$ , the mean values of key variables in the default state versus the non-default state.

Table 3: Mean Values with and without Default

	No Default	Default
$c_t$	0.65	0.64
$c'_t$	1.05	1.05
$h_t$	0.28	0.00
$q_t$	8.46	8.19
$x_t$	0.013	0.072
$x'_t$	0.09	0.00

The most striking result is that the consumption variables exhibit about the same mean in each state. At first glance, this seems to contradict the results of Section 5. This result occurs because of a common pattern that occurs in the run-up to default. Most, but not all, defaults are preceded by increases in both the probability of default and the risky interest rate. For the calibrated model, the former effect is sufficient to induce both an increase in  $c_t$  and a decrease in  $c'_t$ . The expectation of default thus causes most of default's effects to be felt in the period prior to default. As a result, mean consumption is largely unaffected, and housing prices are somewhat close to the average for the no-default state.

Tables 4 and 5 further illustrate this trend. The former redefines the default state as any period where default occurs either in that period, or the next period. The latter redefines the default state as one where default occurs only in the next period. Collectively, they show that as default nears, impatient households respond by increasing their consumption and housing. These effects are more dramatic than those that occur during the default period itself.

Table 4: Mean Values with and without Default (Current or Next Period)

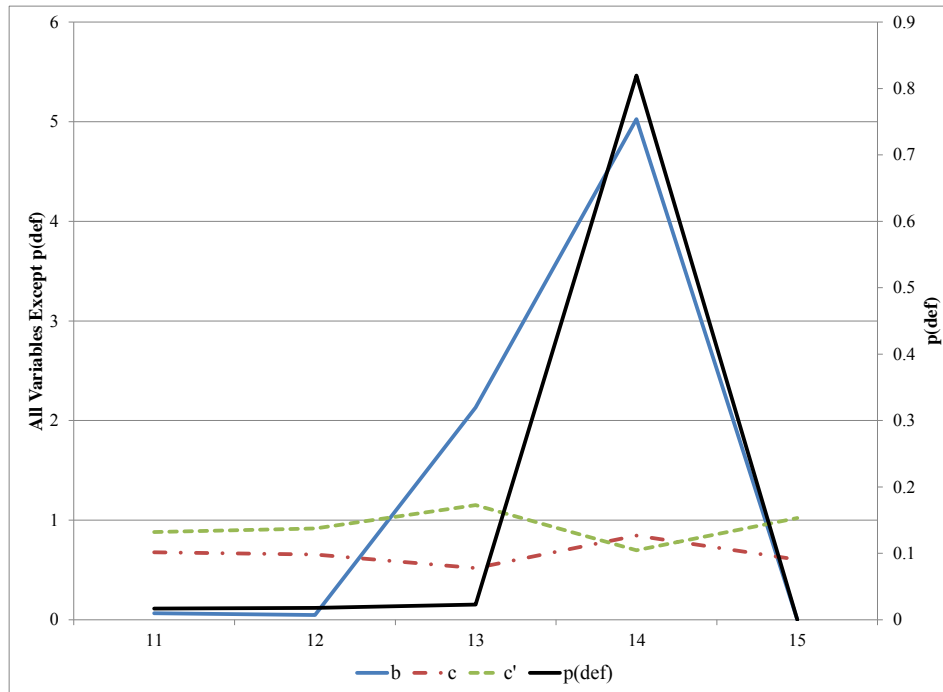
	No Default	Default
$c_t$	0.63	0.68
$c'_t$	1.07	0.99
$h_t$	0.25	0.27
$q_t$	8.46	8.34
$x_t$	0.014	0.039
$x'_t$	0.08	0.09

Table 5: Mean Values with and without Default (Next Period)

	No Default	Default
$c_t$	0.64	0.73
$c'_t$	1.07	0.94
$h_t$	0.22	0.55
$q_t$	8.43	8.49
$x_t$	0.021	0.006
$x'_t$	0.07	0.17

Figure 9 illustrates a fairly common default. For the first 13 periods, default risk is relatively stable at about 2% and the behavior of the variables is, for the most part, uninteresting and not depicted in the chart. In period 14, however, impatient households become highly leveraged as default risk (for period 15) rises to about 80%. In period 15, default does in fact occur.

Figure 9: Sample Default



As the probability of default rises, patient household and impatient household consumption move in opposite directions. Increased debt is used to finance heightened consumption and housing for impatient households, while patient households, anticipating the decline in wealth as a result of default, decrease their consumption as default becomes more likely. These results are consistent with the evidence in Figure 1.

The model thus makes a pair of related predictions. Within a period, as seen in Section 5, default is more likely when demand shocks are high or productivity shocks are low. Default then causes discrete drops in aggregate consumption and housing prices relative to alternative values of the shocks that result in no default. Dynamically, however, these changes tend to be most dramatic just prior to entering the default state.

We now report the converged learning coefficients for each calibration in Table 6. Because agents employ a simple AR(1) specification, these coefficients also show the persistence of each variable:

Table 6: Learning Coefficients for Different Values of  $\chi$ .

	$\chi = 1$	$\chi = 1.1$	$\chi = 1.2$	$\chi = 1.3$	$\chi = 1.4$	$\chi = 1.5$
$a_c$	0.65	0.64	0.64	0.65	0.59	0.57
$b_c$	0.84	0.84	0.82	0.82	0.80	0.80
$a_{c'}$	1.31	1.27	1.22	1.23	1.32	1.35
$b_{c'}$	0.91	0.90	0.88	0.89	0.87	0.89
$a_\pi$	0.97	0.98	0.98	0.97	0.98	0.99
$b_\pi$	0.98	0.98	0.98	0.98	0.98	0.98
$a_q$	9.75	9.22	8.90	9.85	9.28	10.73
$b_q$	0.89	0.88	0.87	0.88	0.86	0.89
$rec_t$	0.81	0.80	0.79	0.81	0.75	0.74
$\Gamma_t$	2.26	2.37	2.38	2.43	2.29	2.20

## 6.1 Monetary Policy

We now consider the impact of the monetary authority responding directly to housing prices by calibrating the model so that  $\phi_q \neq 0$ .<sup>25</sup> Since the bursting of the U.S. housing bubble around 2007, there has been considerable debate over whether the monetary authority should attempt to stabilize housing prices by imposing  $\phi_q > 0$ . The Federal Reserve has resisted such a policy and papers with credit constraints, such as Iacoviello (2005) and Bernanke and Gertler (2001), have generally found little benefit to doing so. The present paper, however, introduces a new mechanism by which asset prices affect monetary policy; thus, we re-examine this issue.

Table 7: Equilibrium Dynamics for Different Values of  $\phi_q$ .

$\phi_q$	Mean( $c_t$ )	SD( $c_t$ )	Mean( $c'_t$ )	SD( $c'_t$ )	Mean( $h_t$ )	Mean( $q_t$ )	Mean( $u_t$ )	Mean( $u'_t$ )	p(def)
-0.20	0.61	0.12	1.12	0.18	0.22	8.54	-0.94	0.03	1.7%
-0.15	0.62	0.12	1.12	0.19	0.24	8.22	-0.91	0.03	2.2%
-0.10	0.63	0.13	1.12	0.19	0.25	8.74	-0.91	0.02	2.8%
-0.05	0.63	0.13	1.11	0.19	0.26	8.92	-0.90	0.01	2.7%
0.00	0.65	0.10	1.05	0.15	0.25	8.44	-0.84	-0.05	10.7%
0.05	0.68	0.15	1.08	0.21	0.25	9.67	-0.81	-0.05	7.9%
0.10	0.68	0.14	1.05	0.20	0.24	9.47	-0.79	-0.08	10.6%
0.15	0.69	0.13	1.03	0.18	0.25	8.56	-0.76	-0.11	12.8%
0.20	0.67	0.14	1.03	0.20	0.27	8.79	-0.79	-0.10	14.9%

Surprisingly, default occurs less often when the monetary authority lowers interest rates in response to higher housing prices. Note that because debt is not indexed in our model, inflation

<sup>25</sup>In addition to choosing  $\phi_q$ , the monetary authority chooses  $\bar{q}$ , the real housing price target. In this section, we set this as the sample mean and discard the first 2000 periods.

volatility contributes to default. Positive demand shocks and lower supply shocks both have the effect of simultaneously raising inflation and lowering housing prices. A negative value of  $\phi_q$  thus has the effect of reinforcing the monetary authority's response to inflation, which results in more stable inflation and less default. A positive value of  $\phi_q$ , however, undermines monetary policy's response to inflation and thus has the opposite effects.

As  $\phi_q$  increases, impatient household utility increases while that of patient households decreases. The former effect is larger than the latter. Thus any social welfare function that has the ordinary property of being indifferent to or penalizing inequality will be maximized for higher values of  $\phi_q$ , despite the higher rates of default.

## 7 Conclusion

This paper adds default to a New-Keynesian model with housing. The previous literature, for the most part, abstracts away from actual mortgage default. We find that allowing for default, as opposed to just the threat of default, has important implications for the model's behavior. Default creates a misallocation of housing that results in a discrete drop in housing prices that amplifies the initial decline that caused default to occur. Furthermore, borrowers increase their consumption due to a beneficial wealth effect and an incentive to substitute toward consumption due to their limited or non-existent ability to purchase housing. However, aggregate consumption, as well as lenders' consumption has the opposite effect.

We also show that the penalty of default matters. If unpaid debt is simply written off, without an accompanying loss of access to either housing or credit markets, then default does not have large discrete effects. It is the lack of borrower access to credit markets that makes default especially interesting. The current paper imposes a one-period penalty, but an interesting extension would be to consider a longer period of exclusion from financial markets. It would also be beneficial to consider other more realistic features of the housing market that we have simplified. We conclude by briefly discussing three. First, it is obviously not the case that all borrowers in the economy

either default or do not. It would thus be of interest to add an idiosyncratic shock to the model that allows for a default rate between 0 and 1. Second, many governments subsidize home ownership through the use of tax incentives. These could be added to the model to examine how they affect aggregate volatility. Finally, this paper treats the housing stock as constant. When default occurs, lower housing prices might incentivize producers to produce less new housing. Endogenizing the housing stock could thus yield larger effects on output than in the present paper.



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