Expectations and the Lucas Critique[1]

The models of the short-run that this class has developed include expectations (of inflation, output, etc.). So far, we have simplified the analysis by imposing very crude methods of expectations formation. These notes seek to more seriously analyze how agents form expectations. How we should assume agents in models form expectations is among the most contentious issues in macroeconomics. It is also often important. We will also examine the Lucas Critique.

A Toy Model

A *toy business cycle model* is one that is not intended to seriously examine business cycles, but is instead a simple model that is used to make a related point. Although we could make many of these same points using the New Keynesian Model, it is much easier to use a toy model. Our first toy model takes the following form:

$$\tilde{Y}_t = -c(i_t - E_{t-1}[i_t]) \tag{1}$$

In this model, deviations of output from potential output ($\tilde{Y}_t \neq 0$) occur if and only if the public is surprised by the interest rate the the Central Bank sets. We assume that if the actual rate (i_t) is less than the expected rate ($E_{t-1}[i_t]$), then output increases above its natural rate. In other words, $c > 0$.[2]

Now assume that the Central Bank uses a policy rule that takes the following form:

$$i_t = n + m\tilde{Y}_t \tag{2}$$

If we assume that the Central bank lowers rates during a recession, then $m > 0$.

We now more carefully examine the expectational term, $E_{t-1}[i_t]$. This represents the public's belief in period $t-1$ of what the interest rate will equal in period t. Unlike the model's other

[1]These are undergraduate lecture notes. They do not represent academic work. Expect typos, sloppy formatting, and occasional (possibly stupefying) errors.

[2]This is known as a *New Classical Model*. Variations of this model were popular during the 1970s.

variables, it is not observable. So we must make some assumption about how the public forms their expectations. We start with a simple case known as *naive expectations.* There are an infinite number of naive expectations. It simply requires that agents do something really simple to form their expectations. For example:

i. $E_{t-1}[i_t] = x$. In words, agents always expect interest rates to equal a constant value. Inserting this and (2) into (1) yields:

$$\tilde{Y}_t = \frac{c(x-n)}{1+cm} \tag{3}$$

The key result here is that the Central Bank can permanently keep output above potential output ($\tilde{Y}_t > 0$). The Central Bank need only choose the appropriate values of m and n that yield such a result. Doing so may have consequences for inflation, but monetary policy now has the ability to affect output (and unemployment) in the long run.

This simple example approximates the state of monetary economics in the 1970s. The Federal Reserve believed that it could keep output above its natural rate in the long run if it was willing to tolerate higher inflation. It thus pursued a policy of "loose money" that attempted to implement this policy. The result was the "Great Inflation" with inflation reaching 14% by 1980.

The Great Inflation did not yield permanently lower unemployment or higher output. The problem is that policy makers were assuming that the public was too stupid, forming expectations like those in this example. Economists such as Robert Lucas and Milton Friedman thus proposed an alternative known as *rational expectations.*

ii. Rational expectations imply that agents form mathematically optimal expectations. in other words, when forming expectations, agents optimize just like they do when choosing labor demand or consumption. Consider $(i_t - E_{t-1}[i_t])$, the difference between actual and expected inflation. Under rational expectations, agents may choose a rule that minimizes the expected sum of squared expectational errors.

Suppose that the public knows that the Central Bank uses (2) to set interest rates. Under rational expectations, they use this knowledge to form optimal expectations. It must be true that:

$$E_{t-1}[i_t] = n + mE_{t-1}[\tilde{Y}_t] \tag{4}$$

Notice that under rational expectations, the public is smart enough to know that if $\tilde{Y}_t = 0$, then the Central Bank chooses $i_t = n$. Under naive expectations they, possibly mistakenly, thought that the Central bank would set $i_t = x$.

We still must deal with the public's expectation in period $t-1$ of $\tilde{Y}_t$. Let us guess that $E_{t-1}[\tilde{Y}_t] = 0$. This may seem to be like naive expectations instead of rational expectations. We must therefore, *verify* that this is rational later on.

Inserting $E_{t-1}[i_t] = n$ and (2) into (1) yields:

$$\tilde{Y}_t = 0 \tag{5}$$

Output therefore always equals potential output. While this isn't interesting as an explanation of business cycles, our toy model does illustrate the impact of rational expectations versus naive expectations. We can also verify that it is rational to expect the output gap to always equal zero because (5) shows that is true.

Under rational expectations, the Central Bank cannot affect output in the long run (or the short run in this model). Using (2), note that $i_t = n$. The Central bank is still free to choose n. recall that lower interest rates require more money and higher prices. If a Central Bank thus mistakenly believes that it can permanently affect output by tolerating high inflation, then such a policy will achieve high inflation but without the hoped for increase in output.

This story approximates the experience of the United States in the 1970s. It helps explain why contemporary macroeconomics takes expectations formation very seriously.

The Lucas Critique

To examine the Lucas Critique, we consider another, slightly more complicated, toy model. Suppose that:

$$Y_t = a - c(i_t - E_{t-1}[i_t]) + u_t \tag{6}$$

where u_t is an exogenous shock. Further suppose that the Central Bank uses the following rule to set interest rates:

$$i_t = d \tag{7}$$

If the public forms rational expectations, then they use (7) to form expectations of i_t.

$$E_{t-1}[i_t] = d \tag{8}$$

Inserting (8) into (6) yields:

$$Y_t = (1 + cd) - ci_t + u_t \tag{9}$$

Now suppose that a Central Bank runs a regression based on (9) and correctly estimates the constant term and the coefficient on i_t.

$$Y_t = \alpha + \beta i_t + e_t \tag{10}$$

Consider the following exercise:

i. The Central bank interprets the regression coefficients (α and β) as exogenous and time-invariant parameters.

2. The Central Bank then decides that it wishes to increase average output. It thus decides to lower d by 1%. It then expects that output each period output will increase by β% (or c% if (9) is measured without error) because it believes that α is exogenous.

3. α is not, however, actually exogenous. When d decreases by 1% so does the true value of α (as seen in (9)). This effect perfectly cancels out that from #2. To see this, solve for the model's steady state under the assumption that the average value of u_t is zero:

$$Y = a \tag{11}$$

This is an example of the *Lucas Econometric Critique*. Under rational expectations, expectations depend on policy parameters. The resulting regression coefficients thus also depend on these policy parameters. If we change the policy, then we change the regression coefficients. **Under these circumstances, we cannot use regression coefficients to analyze the effect of changing policies**.

The Lucas Critique has many applications outside macroeconomics. Consider the following example:

1. The City of Lewiston fines people for $20 for illegally parking outside City Hall.

2. Lewiston notices that it collects $1000 per year from these fines.

3. Lewiston uses this simple statistical relationship to conclude that if it raises the fine to $20,000,000, it will then collect $1 billion in revenue that can then be used to build a monorail.

Such a policy would almost surely fail. By changing the policy, drivers will change their expectations. They will now never risk a parking ticket and revenue will thus likely fall to zero. By changing the policy, Lewiston changes the underlying statistical relationship. The old relationship cannot be used to forecast the effect of the policy change.

Learning

It is fair to criticize naive expectations for people too stupid. It is also fair, however, to criticize rational expectations for making people too smart. Under rational expectations, agents know the true model and use it to form expectations. This is implausible, however, because there is no agreement among macroeconomists over which is the best model. A third approach is known as learning. It is motivated by the idea of *cognitive consistency*, that agents in the model should be about as smart as the person modeling them. If an economist were asked to estimate a future macroeconomic variable, she would very likely run some regressions. So learning assumes that households run regressions to form expectations. Consider the following illustrative example:

1. A household wishes to forecast future consumption so that they can better smooth their consumption. So they regress C_t on perhaps i_{t-1} and C_{t-1}.

2. The household then uses this estimate to form expectations. Suppose for example that they obtain the following regression result:

$$C_t = \alpha + \beta_1 C_{t-1} + \beta_2 i_{t-1} + u_t \tag{12}$$

It then follows that:

$$E_t[C_{t+1}] = \alpha + \beta_1 C_t + \beta_2 i_t \tag{13}$$

3. The household then uses (13) to choose C_t. This generates a new observation.

4. The household then adds this observation to its data and updates its regression.

Using learning yields many interesting results to common macroeconomic models. We will not derive these. But a few important results follow:

i. Econometric models are estimated with error. Using learning thus adds additional uncertainty into a model, much like adding an extra shock. Models under learning are thus more volatile than under rational expectations.

ii. Because regressions use old data, the current state of the economy depends more on the past than under rational expectations. As a result, under learning macroeconomic shocks have effects for longer periods of time and it takes longer for the peak effect to occur. It is said that learning adds persistence to models. Recall that our New Keynesian model lacked persistence. Learning helps fix this weakness.

iii. Recall the New Keynesian Phillips Curve:

$$\pi_t = \pi^e_{t+1} + \bar{v}\tilde{Y}_t + u_t \tag{14}$$

Under learning, when the public comes to expect high inflation, it takes a long time for this belief to change because learning uses old data. Note from (14) that expected inflation causes current inflation. This reinforces expectations of high future inflation. It is thus very important that Central Banks prevent high inflation from occurring in the first place.