Endogenous Growth and Household Leverage

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Abstract

We add households with heterogeneous discount factors, and who are subject to credit constraints to a research and development (R&D) based endogenous growth model. We find that altering borrowers' access to credit often has profound implications on the steady-state growth rate. The direction and magnitude of this effect depends on our assumptions about households' preferences over labor supply. When labor supply is highly elastic and households do not try to smooth their labor supply between labor that produces output and R&D labor, the annual growth rate decreases from 11.6% to approximately zero as the debt-to-capital ratio rises from 0 to 1.38. However, if households instead have a strong preference for smoothing their labor supply between production and R&D, then growth increases from 2.91% to 3.83% as the debt-to-capital ratio rises from 0 to 1.55. In both cases, as labor supply becomes less elastic, these effects become weaker.

JEL Classification: E21, E22, E44, O40

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1 Introduction

Over the past decade, a considerable literature has examined the impact of assuming that households can only borrow up to a fraction of their asset holdings. The analysis of these credit constraints has mostly focused on how they allow demand shocks to be amplified and propagated through the financial accelerator effect.[1] In this paper, we add credit constraints with heterogeneous households (differentiated by discount factor) into a research and development (R&D) based endogenous growth model similar to Jones (1995).[2] Households must choose how much labor to supply to the productive sector and how much labor to supply to R&D, the latter of which determines the economy's steady-state growth rate of total factor productivity (TFP), output, and consumption. We show that varying the maximum household leverage ratio often has enormous effects on the steady-state growth rate.

The relationship between access to credit and growth depends primarily on two parameters.[3] The first determines how much households wish to smooth their labor supply between productive labor and R&D. The second is the Frisch elasticity of labor supply. Because the appropriate value of both parameters is unclear, we reports several combinations.[4] In one example, we assume both that households have little desire to smooth their labor supply between R&D and productive labor and that aggregate labor supply is highly elastic. Here, the creditors (patient households) are wealthier than borrowers (impatient households). When households do not have access to credit, the steady-state growth rate is 11.6% per year. After increasing

[1] See Kiyotaki and Moore (1997) and Iacoviello (2005).

[2] For a more complete discussion of models with growth driven by endogenous technological change, see Romer (1990), Grossman and Helpman (1991a,b,c), and Aghion and Howitt (1992).

[3] We use the terms access to credit, debt-to-capital ratio, leverage ratio, and loan-to-value ratio interchangeably. While they do represent slightly different measures in the data, in our model, they are synonymous.

[4] See Chetty et al. (2011) for further discussion of the controversy over the correct calibration of the Frisch elasticity of labor supply. Highly elastic labor supply tends to allow macroeconomic theory models to better fit the data while microeconometric studies tend to suggest less elasticity. The reader may reasonably conclude, however, that the effect of access to credit on growth is implausibly high when labor supply is very elastic in our model and that less elastic labor supply thus yields better fit for our model. Because our model introduces households' desire to smooth their labor supply across types as a new parameter, we are unaware of any existing work that attempts to calibrate its value.

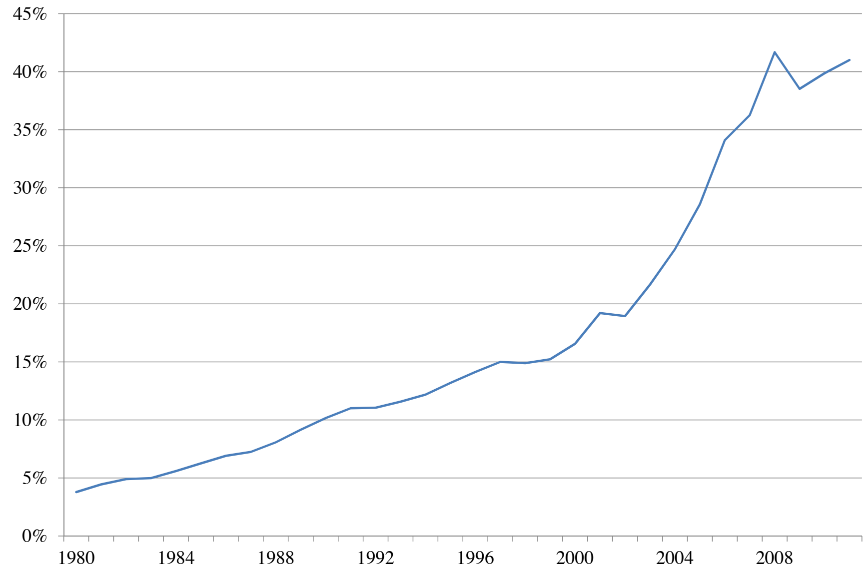
the amount impatient households are allowed to borrow to the point where their debt-to-capital ratio equals 1, growth falls to 3.03%. By the time the leverage ratio reaches 1.4, growth is almost zero. This result occurs because more debt increases the wealth of creditors. The creditors respond by substituting toward leisure and away from labor supply, including R&D labor, which causes a reduction in growth. When we allow labor supply to be less elastic, the effect is smaller, but still meaningful for even very inelastic labor supply.

We also find cases where more debt increases growth. Assuming that households prefer to smooth their labor between production and R&D and that labor supply is relatively elastic, growth is now driven more by the R&D choices of borrowers. As the borrowers become more indebted, they supply more of all types of labor, including R&D. The steady-state growth rate rises from 2.91% without debt to 3.00% when the leverage ratio equals 1. Furthermore, the steady-state growth rate increases to 3.83% when the leverage ratio equals 1.55. Again, the magnitude of these effects diminishes as labor supply becomes less elastic.

In much of the credit constraints literature, the leverage ratio is treated as a constant determined exogenously and representing the ability of lenders to recover collateral from borrowers in the case of default.[5] We argue that it is important to examine how a variable leverage ratio affects growth because there is ample evidence that the leverage ratio is neither constant nor immune to policy. Figure 1 plots household debt as a percentage of physical capital for the United States between 1980 and 2011:

[5]This interpretation dates from the seminal work of Kiyotaki and Moore (1997).

Figure 1: U.S. Household Debt to Capital

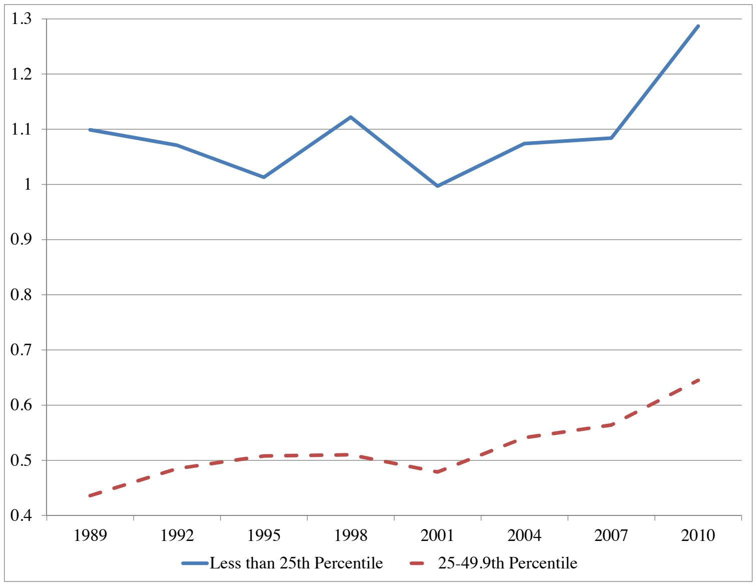


Source: Federal Reserve Economic Data, St. Louis Federal Reserve Bank

Figure 1 illustrates that access to credit has both been trending upward and has been volatile around its trend. However, because these data are aggregate, they do not compare directly to the leverage ratio in our model, which applies only to specific types of households (impatient).

Further decomposing the data by household net worth, Figure 2 below shows the leverage ratio (total debt-to-assets) for households in the lowest two percentiles of net worth from 1989 to 2010:

Figure 2: Leverage Ratio by Percentile of Net Worth



Source: Federal Reserve Board of Governors Survey of Consumer Finances Table 12

Figure 2 provides additional evidence that the leverage ratio is variable. In addition, Figure 2 shows that, for the poorest households, the leverage ratio often exceeds 1. In our model, borrowers are almost always poorer than creditors, and we identify cases where allowing borrowers to go "underwater" (a debt-to-capital ratio greater than one) has important implications for growth. When households have little desire to smooth their labor supply across productive labor and R&D, allowing underwater borrower yields especially low growth rates, often near zero. When households have a strong desire to smooth labor across types, however, allowing underwater borrowing maximizes growth.

There is also ample anecdotal evidence of policymakers attempting to influence leverage ratios. The existence of the mortgage giants Fannie Mae and Freddie Mac, government sponsored enterprises in the United States, may fairly be viewed as a policy attempt to facilitate greater access to mortgage debt. Provisions in the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010 attempt to induce lower leverage among American firms, and the

Basel Accords may be viewed as an international effort to minimize debt-to-capital ratios in the banking sector. We thus consider it important to understand how changing leverage ratios affect growth.

Most of our analysis focuses on the model's steady state. We also linearize the model and analyze its dynamics around the steady state. We find that the model is always determinate in that a unique stationary rational expectations equilibrium exists. Notably, the model's response to a temporary shock to the leverage ratio does not significantly depend on whether households prefer to smooth their labor supply across types. In all cases, borrowers respond to more credit by temporarily increasing their capital, consumption, and output, while lenders have the opposite response. We do not find that temporary shocks have important long-term effects on the level of total factor productivity (TFP).

This is the first paper to combine an endogenous growth model with a R&D sector and a model that includes heterogeneous households and credit constraints. There is, however, a small literature that incorporates collateral constraints, using the Kiyotaki and Moore (1997) framework in growth models. Several papers use liquidity constraints in an endogenous growth model where growth is determined by capital accumulation instead of technological advancement through R&D. Jappelli and Pagano (1994) find that credit constraints increase the savings rate of households, which leads to a higher growth rate. Amable et al. (2004) use a model in which net worth determines borrowing capacity and examine the impact of a change in the interest rate on growth. They conclude that when the interest rate increases, growth decreases due to the negative impact on retained earnings and the negative leverage effect. Finally, Bencivenga and Smith (1993) consider a model in which all investment activities are externally financed and find that an increase in capital production technology magnifies the negative effects of credit rationing and reduces growth. These papers, however, either do not examine the effects of changes in leverage on growth, or do not find that the leverage ratio has a significant impact on growth rates. In contrast, we find that the effects of altering the leverage ratio are often dramatic.

Our paper also contributes to the mostly empirical literature on growth and financial development. There is considerable debate over whether growth causes financial development or vice-versa. Levine (2005) provides an overview and argues that the bulk of the evidence suggests that greater financial development does promote growth. Although it is not obvious how to equate leverage to general financial development, our paper presents a novel channel through which changes in the financial sector affect growth.

This paper is organized as follows. Section 2 outlines the theoretical model. Section 3 shows the steady state results and the impact of varying access to credit. Section 4 analyzes the model's dynamics around the steady state. Section 5 concludes.

2 The Model

Following the related literature on credit constraints, we assume that a set of patient households, with relatively high discount factors, lends to a set of impatient households. Also consistent with the previous literature, we impose a collateral constraint on the impatient households who borrow from the patient households. Our contribution is to include a R&D-based endogenous growth sector similar to Jones (1995). Both types of households are infinitely lived and exist on a continuum on the unit interval. Patient and impatient households produce, consume, and supply labor to the R&D and the production sector. Technology used by the lenders and borrowers is the same and evolves according to a recursive structure:

$$A_{t+1} - A_t = \mu(L_{a,t}^{\lambda} + L_{a,t}^{'\lambda}) \tag{2.1}$$

where A_t represents technology in time t, and $L_{a,t}$ and $L_{a,t}^{'}$ are labor supplied to the technology sector by lenders and borrowers respectively. For each simulation, we select a value of μ that yields a growth rate of approximately 3% when the impatient households' debt-to-capital ratios equal one.

Patient households are savers and lend to the impatient households. They maximize expected lifetime utility, which is a function of consumption (c_t), hours worked for R&D ($L_{a,t}$), and hours worked for production ($L_{y,t}$):

$$\underset{c_t, L_{a,t}, L_{y,t}, k_{t+1}}{\text{Max}} \qquad E_0 \sum_{t=0}^{\infty} \beta^t \left(lnc_t - \frac{\chi (L_{a,t}^{\epsilon} + L_{y,t}^{\epsilon})^{\frac{\eta}{\epsilon}}}{\eta} \right)$$

where the expectation operator is E_0, the discount factor is β (calibrated at a value of 0.99), χ is the weight placed on the disutility associated with supplying labor, ϵ is a parameter that dictates the substitutability between labor supplied to R&D and production, and η is the inverse Frisch elasticity of labor supply. They are subject to a budget constraint:

$$c_t + k_{t+1} + b_t \leq R_{t-1} b_{t-1} + Y_t + (1 - \delta) k_t$$

where c_t is patient household consumption, k_t is patient household capital stock, δ is the depreciation rate, and Y_t is patient household output. The borrower and lender relationship between the two households is defined by their relative discount factors. The discount factor of the impatient households is less than that of patient households. As a result, the impatient households are the borrowers and patient households are the lenders in the model. The loan structure is such that in time t, a loan of amount b_t is made from the patient households to the impatient households. In the subsequent period, the impatient households pay off the debt at an interest rate of r_t, where $R_t = 1 + r_t$. Therefore, in time t, the value of the loan plus interest from time $t-1$, $R_{t-1}b_{t-1}$, is an additional source of income for the patient households.

In addition, patient households produce according to the following constant returns to scale (in labor and capital, given technology) production function:

$$Y_t = (A_t Z_t L_{y,t})^{\alpha} k_t^{(1-\alpha)}$$

There are two components of TFP: a permanent component (A_t) and a transitory component (Z_t) that is subject to random productivity shocks.

The first order conditions for the patient households are:

$$E_t \left[\frac{1+g_{t+1}}{\tilde{c}_t U_t} \right] = \left[\frac{\beta}{\tilde{c}_{t+1} U_{t+1}} (1 - \delta + Z_{t+1}(1-\alpha) L_{y,t+1}^{\alpha} \tilde{k}_{t+1}^{-\alpha}) \right] \tag{2.2}$$

$$E_t \left[\frac{1+g_{t+1}}{c_t \tilde{U}_t} \right] = E_t \left[\frac{\beta R_t}{\tilde{c}_{t+1} U_{t+1}} \right] \tag{2.3}$$

$$\frac{\alpha Z_t L_{y,t}^{\alpha-1} \tilde{k}_t^{\,1-\alpha}}{\tilde{c}_t U_t} = \chi (L_{a,t}^{\epsilon} + L_{y,t}^{\epsilon})^{\frac{\eta}{\epsilon}-1} L_{y,t}^{\epsilon-1} \tag{2.4}$$

$$\begin{aligned} \chi (L_{a,t}^{\epsilon} + L_{y,t}^{\epsilon})^{\frac{\eta}{\epsilon}-1} L_{a,t}^{\epsilon-1} = E_t \Bigg[& \frac{\beta \lambda \mu \alpha Z_{t+1} L_{a,t}^{\lambda-1} L_{y,t+1}^{\alpha} \tilde{k}_{t+1}^{1-\alpha}}{\tilde{c}_{t+1}(1+g_{t+1}) U_{t+1}} \\ & + \beta \chi L_{a,t+1}^{\epsilon-1} (L_{a,t+1}^{\epsilon} + L_{y,t+1}^{\epsilon})^{\frac{\eta}{\epsilon}-1} \left(\frac{L_{a,t}}{L_{a,t+1}} \right)^{\lambda-1} \left(\frac{1+g_{t+2}}{1+g_{t+1}} \right) \Bigg] \end{aligned} \tag{2.5}$$

where equation 2.2 is patient household demand for capital, equation 2.3 is the Euler equation, equation 2.4 is patient household labor supply to the production sector, and equation 2.5 is patient household labor supply to the R&D industry. Consumption and capital have been detrended such that $\tilde{c}_t = \frac{c_t}{A_t}$, $\tilde{k}_t = \frac{k_t}{A_t}$, and $\tilde{b}_t = \frac{b_t}{A_t}$.

To see the intuition behind equation 2.5, the supply of R&D labor, consider a case where $L_{a,t}$ changes by one unit such that $dL_{a,t} = 1$. Solving for $dL_{a,t+1}$ yields:

$$dL_{a,t+1} = -\left(\frac{L_{a,t}}{L_{a,t+1}}\right)^{\lambda-1}\left(\frac{1+g_{t+2}}{1+g_{t+1}}\right) \tag{2.6}$$

where

$$g_{t+1} = \frac{A_{t+1}-A_t}{A_t} = \mu(L_{a,t}^{\lambda} + L_{a,t}^{'\lambda}) \tag{2.7}$$

In order for A_{t+2} to remain unchanged, if labor is increased by one unit in time t, labor in time $t+1$ changes by $dL_{a,t+1}$. Equation 2.5 equates the benefits and costs of changing $L_{a,t}$ and $L_{a,t+1}$ by these amounts.

Impatient households, denoted by the prime symbol, work, consume, produce, and borrow from the patient households. They maximize expected lifetime utility:

$$\underset{c_t^{'},L_{a,t}^{'},L_{y,t}^{'},k_{t+1}^{'}}{\text{Max}} \qquad E_0 \sum_{t=0}^{\infty} \beta^{'t}\left(lnc_t^{'} - \frac{\chi(L_{a,t}^{'\epsilon} + L_{y,t}^{'\epsilon})^{\frac{\eta}{\epsilon}}}{\eta}\right)$$

subject to a budget constraint:

$$c_t^{'} + k_{t+1}^{'} + R_{t-1}b_{t-1} = b_t + Y_t^{'} + (1-\delta)k_t^{'}$$

and a credit constraint:

$$R_t b_t \leq m_t k_{t+1}^{'}$$

where m_t evolves according to an AR(1) process such that:

$$m_t = m_{t-1}^{\rho_m} e_{m,t} \tag{2.8}$$

and $e_{m,t}$ is a random shock. The detrended credit constraint is given by:

$$\tilde{b}_t \leq \frac{(1+g_t)m_t\tilde{k}'_{t+1}}{R_t} \tag{2.9}$$

The collateral constraint on impatient households requires that the value of their debt plus interest cannot exceed the amount of recoverable future capital. In this setup, the threat of default matters, but borrowers are not allowed to actually be insolvent.[6] We interpret the variable m_t as representing access to credit. Consider the credit constraint evaluated at the steady state. Provided that the impatient household discount rate is less than the patient household discount rate ($\beta' < \beta$), at the steady state, the collateral constraint will be binding:

$$\tilde{b} = \frac{(1+g)m\tilde{k}'}{R}$$

At the steady state, $R = \frac{(1+g)}{\beta}$ and β is calibrated to be 0.99. Using an approximation where $\beta = 1$, plugging in the value for R and rearranging yields:

$$m = \frac{\tilde{b}}{\tilde{k}'} \tag{2.10}$$

which is the debt-to-capital ratio. The variable m can be interpreted as a leverage ratio. It is analogous to capital requirements for firms or loan-to-value (LTV) ratios for households. In the credit constraints literature, m is both treated as a constant and often interpreted to be one minus the cost of recovering collateral in the event of default. This latter interpretation constrains m to be less than one. As discussed in Section 1, however, the data show that m is both volatile and, for subsets of households, possibly greater than one. Thus, we simply interpret m as the exogenous level of access to credit and only constrain it to be non-negative.

Impatient households also produce using their own labor and capital:

[6] Marshall and Shea (2014) relax this assumption, allowing agents to explicitly default and show that this causes a discrete drop in asset prices and output.

$$Y_t^{'} = (A_t Z_t L_{y,t}^{'})^{\alpha} k_t^{'(1-\alpha)}$$

The first order conditions for the impatient households are:

$$E_t\left[\frac{(1+g_{t+1})}{\tilde{c}_t^{'}}\right] = E_t\left[\frac{\beta^{'}(1-\delta+Z_{t+1}(1-\alpha)L_{y,t+1}^{'\alpha}\tilde{k}_{t+1}^{'-\alpha})}{\tilde{c}_{t+1}^{'}} + m\tilde{\gamma}_t(1+g_{t+1})\right] \tag{2.11}$$

$$\tilde{\gamma}_t R_t + E_t\left[\frac{\beta^{'}R_t}{\tilde{c}_{t+1}^{'}(1+g_{t+1})U_{t+1}}\right] = \frac{1}{\tilde{c}_t^{'}U_t} \tag{2.12}$$

$$\frac{\alpha Z_t L_{y,t}^{'(\alpha-1)}\tilde{k}_t^{'(1-\alpha)}}{\tilde{c}_t^{'}U_t} = \chi(L_{a,t}^{'\epsilon} + L_{y,t}^{'\epsilon})^{\frac{\eta}{\epsilon}-1}L_{y,t}^{'(\epsilon-1)} \tag{2.13}$$

$$\begin{aligned}\chi(L_{a,t}^{'\epsilon} + L_{y,t}^{'\epsilon})^{\frac{\eta}{\epsilon}-1}L_{a,t}^{'(\epsilon-1)} = E_t\Bigg[&\frac{\beta^{'}\lambda\mu\alpha Z_{t+1}L_{a,t}^{'(\lambda-1)}L_{y,t+1}^{'\alpha}\tilde{k}_{t+1}^{'(1-\alpha)}}{\tilde{c}_{t+1}^{'}(1+g_{t+1})U_{t+1}} \\ &+ \beta^{'}\chi L_{a,t+1}^{'(\epsilon-1)}(L_{a,t+1}^{'\epsilon} + L_{y,t+1}^{'\epsilon})^{\frac{\eta}{\epsilon}-1}\left(\frac{L_{a,t}^{'}}{L_{a,t+1}^{'}}\right)^{\lambda-1}\left(\frac{1+g_{t+2}}{1+g_{t+1}}\right)\Bigg]\end{aligned} \tag{2.14}$$

where equation 2.11 is impatient household demand for capital, equation 2.12 is the Euler equation, equation 2.13 is impatient household labor supply to the production sector, and equation 2.14 is impatient household labor supply to the R&D industry. The Lagrange multiplier on the credit constraint is detrended by $\tilde{\gamma}_t = \gamma_t A_t$.

Other equations in the system include:

$$\tilde{Y}_t = L_t^{\alpha}\tilde{k}_t^{1-\alpha} \tag{2.15}$$

$$\tilde{Y}_t' = L_t'^{\alpha} \tilde{k}_t'^{(1-\alpha)} \tag{2.16}$$

$$\tilde{c}_t + \tilde{k}_{t+1}(1+g_{t+1}) + \tilde{b}_t \leq \frac{R_{t-1}\tilde{b}_{t-1}}{(1+g_t)} + \tilde{Y}_t + (1-\delta)\tilde{k}_t \tag{2.17}$$

$$\tilde{c}_t' + \tilde{k}_{t+1}'(1+g_{t+1}) + \frac{R_{t-1}\tilde{b}_{t-1}}{(1+g_t)} \leq \tilde{b}_t + \tilde{Y}_t{}' + (1-\delta)\tilde{k}_t{}' \tag{2.18}$$

where equations 2.15 and 2.16 are the detrended production function for the patient and impatient households respectively, and equations 2.17 and 2.18 are the detrended budget constraints for the patient and impatient households respectively. Finally, we assume the transitory part of TFP follows an AR(1) process according to equation 2.19 and that the shock to preferences (U_t) is iid with mean equal to one.

$$Z_t = Z_{t-1}^{\rho_a} e_{Z,t} \tag{2.19}$$

3 The Balanced Growth Path and Leverage

This section examines the model's steady state and shows that the effects of varying the leverage ratio on the growth rate may be in either direction, depending on the calibration, and that they are potentially large. Two parameters are critical: ϵ and η. When ϵ is close to one, suggesting that households have little desire to smooth their labor supply, higher leverage reduces growth. High values of ϵ, which implies a desire to smooth labor supply, however, cause higher leverage ratios to increase growth. The magnitude of the effects are large when η is low (suggesting elastic labor supply) and smaller when it is high. Because ϵ is novel to our paper and the correct calibration of η is controversial, we consider several alternate calibrations.

We also consider the optimal leverage ratio, calibrated using a social welfare function evaluated at the steady state. These results are sensitive to both our calibration and our choice of social welfare functions, specifically whether we use a Utilitarian function that generally tracks the utility of patient households or a Rawlsian function that generally tracks the utility of impatient households. We find cases where optimality occurs at $m = 0$, suggesting that allowing any debt is welfare reducing and cases where the optimal leverage ratio is greater than one, which implies that households should be allowed to go underwater on their debt.

As far as possible, we follow the related literature in calibrating our model. We set $\alpha = 2/3$, a standard value for the Cobb-Douglas production function. We set $\beta = 0.99$, implying a real interest rate of 4% for quarterly data. We set $\beta' = 0.95$, and, as in Jones (1995), we set $\lambda = 1$.[7] We set $\delta = 0.025$, another standard value. For each simulation, we fix μ so that the steady-state growth rate for $m = 1$ is near 3%. We then examine the effects of changing m. Table 1 below summarizes the baseline calibration.

Table 1: Calibration

α	labor's share in production function	0.67
β	patient households' discount factor	0.99
β'	impatient households' discount factor	0.95
χ	weight on labor supply in utility function	1
λ	returns to scale on R&D	1
δ	capital depreciation rate	0.025
β'	impatient households' discount factor	0.95

3.1 Simulation 3.1: $\epsilon = 1.1$, $\eta = 1.1$, $\mu = 0.01222$

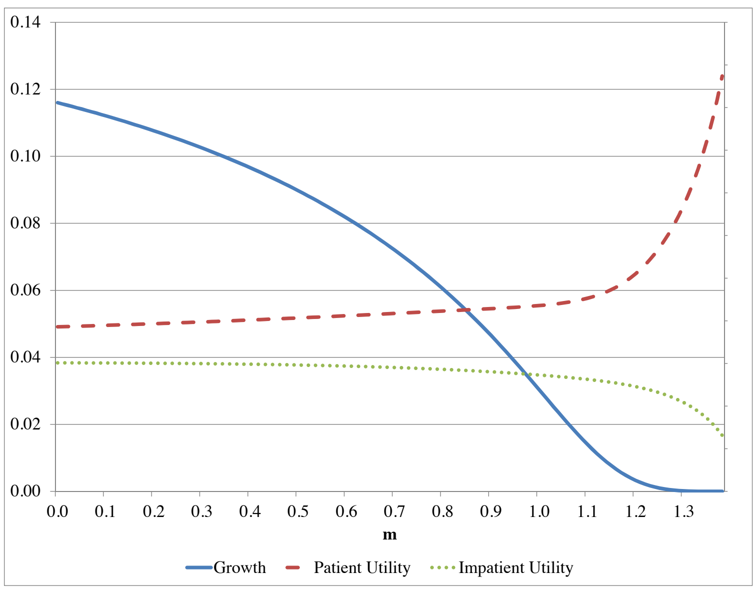
In this simulation, ϵ is low suggesting that households have little taste for smoothing their labor supply across types. In addition, labor supply is highly elastic ($\eta = 1.1$). The key implication of having a low ϵ is that impatient households, having a lower discount factor, supply very little R&D. As a result, the R&D decisions of patient households almost entirely

[7]As $\beta' \to \beta$ debt approaches zero and the effects changing m also go to zero.

drive growth.

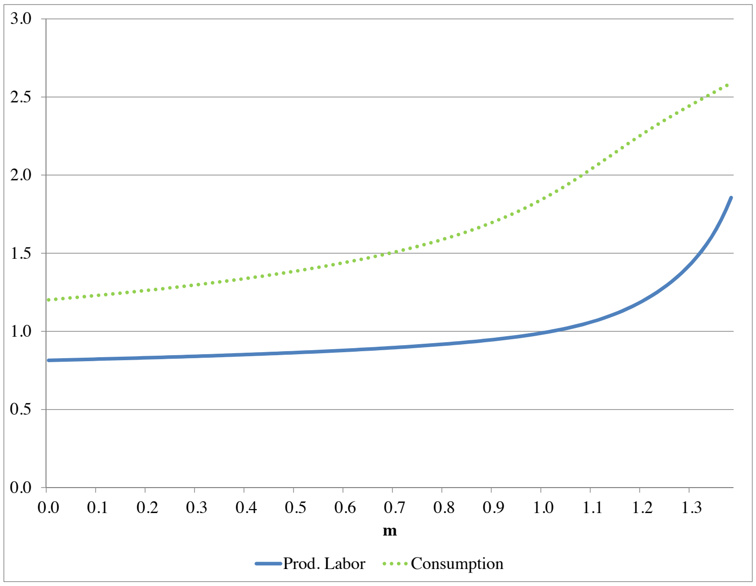
For all of our simulations, we choose μ to yield about a 3% growth rate at $m = 1$. In this simulation, growth equals 11.60% when $m = 0$. Here, there is no interaction between the two types of households, except for an uncorrected positive externality where each of their R&D benefits the other. As shown in Figure 3, the growth rate continues to fall as m increases, reaching 3.03% at $m = 1$ and it is near zero by $m = 1.38$

Figure 3: Effects of Varying m on Growth and Utility



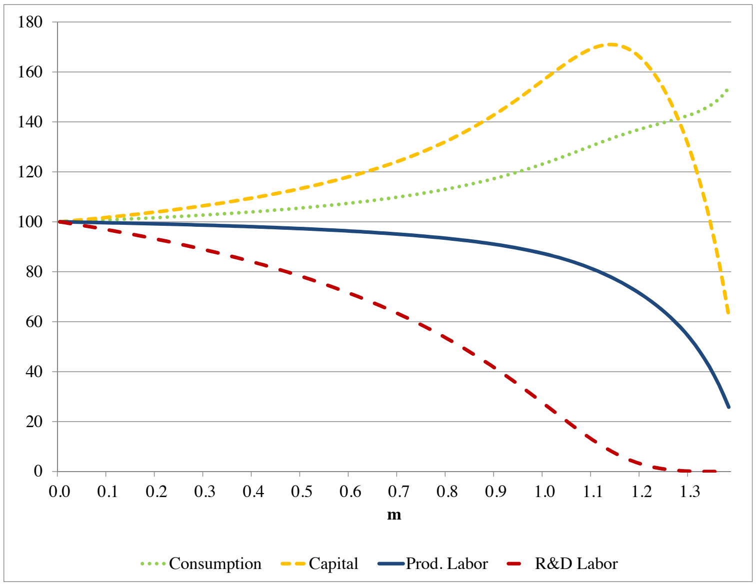
The effect of a change in access to credit on growth in this simulation is dramatic. Growth becomes virtually nonexistent when the impatient households are allowed to be highly leveraged. At the steady state, higher leverage transfers wealth from impatient households to patient households. Impatient households respond by increasing their productive labor. But because they supply almost no R&D, this has virtually no impact on growth. Figure 4 summarizes the impact of a change in access to credit on impatient households.

Figure 4: Effects of Varying m on Impatient Households



As m increases, patient households become wealthier. They respond by substituting away from both types of labor towards leisure. Because they drive growth, the reduction in their R&D causes the aggregate growth rate to collapse. For very high leverage ratios, patient households are content to live almost entirely on debt payments while supplying little labor and causing very low growth. Figure 5 shows the effects of a change in m for the patient households. The values for all variable are normalized so that they equal 100 when $m = 0$.

Figure 5: Effects of Varying m on Patient Households



We now evaluate the welfare implications of different values of m using the steady-state levels of utility for each type of household. Iterating the utility functions forward at the steady state and taking infinite geometric series yields:

$$U = \frac{1}{1-\beta}\left[ln(c) - \chi\frac{(L_a^{\epsilon} + L_y^{\epsilon})^{\frac{\eta}{\epsilon}}}{\eta} + \frac{ln(g)}{1-\beta}\right] \tag{3.1}$$

$$U^{'} = \frac{1}{1-\beta}\left[ln(c^{'}) - \chi\frac{((L_a^{'})^{\epsilon} + (L_y^{'})^{\epsilon})^{\frac{\eta}{\epsilon}}}{\eta} + \frac{ln(g)}{1-\beta}\right] \tag{3.2}$$

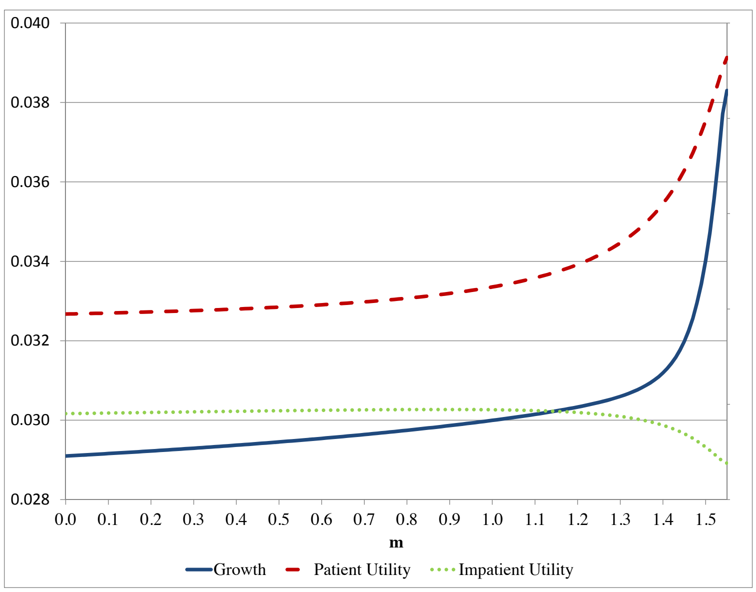
These utility levels are included in Figure 3. Steady-state utility is maximized for impatient households at $m = 0$ because their wealth and growth are highest and for patient households at $m = 1.38$ (the largest value of m in this calibration). A Utilitarian social welfare function based on steady-state utilities is maximized at $m = 1.38$, while a Rawlsian is welfare function is maximized at $m = 0$, because it perfectly tracks impatient utility.

This simulation illustrates how higher leverage can cause a dramatic decline in growth. Increasing access to credit causes both lower growth and much higher levels of income inequality.

3.2 Simulation 3.2: $\epsilon = 10$, $\eta = 1.1$, $\mu = 0.0042$

In this case, we continue to assume that labor supply is highly elastic. But now, we set $\epsilon = 10$ so that households try to smooth their labor supply between the two types. We also lower μ in order to keep growth rates close to 3% at $m = 1$. Figure 6 shows the effects of varying m on the growth rate and utility.

Figure 6: Effects of Varying m on Growth and Utility

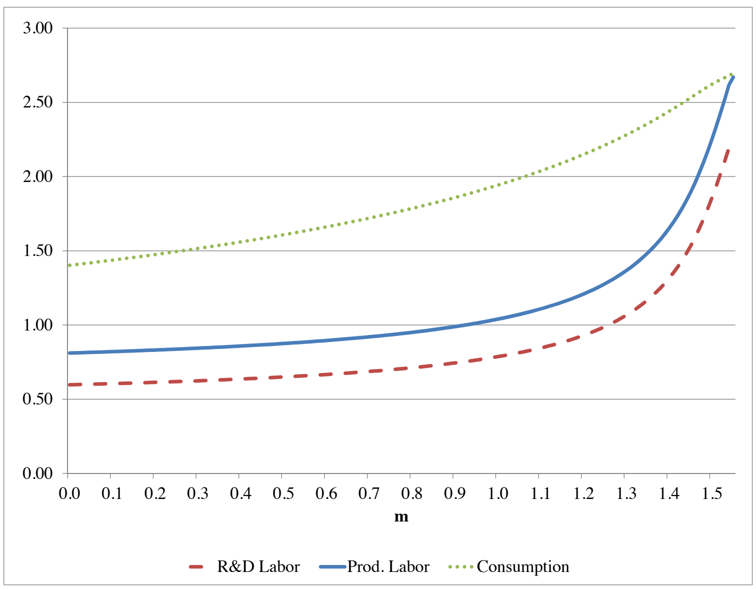


When $m = 0$, the annualized growth rate is 2.91%. As m rises to one, the growth rate exhibits a small but important increase to 3.00%. As m increases to 1.55, however, the increase in the growth rate accelerates, rising to 3.83%.

By assuming that households wish to smooth their labor across types, we now induce impatient households to supply significant levels of R&D. As m increases and wealth is transferred from impatient households to patient households, the former now respond by increasing their supply of both types of labor while the latter respond by decreasing both of their labor supplies. In this simulation, the effect on impatient households is dominant and growth increases along with m.

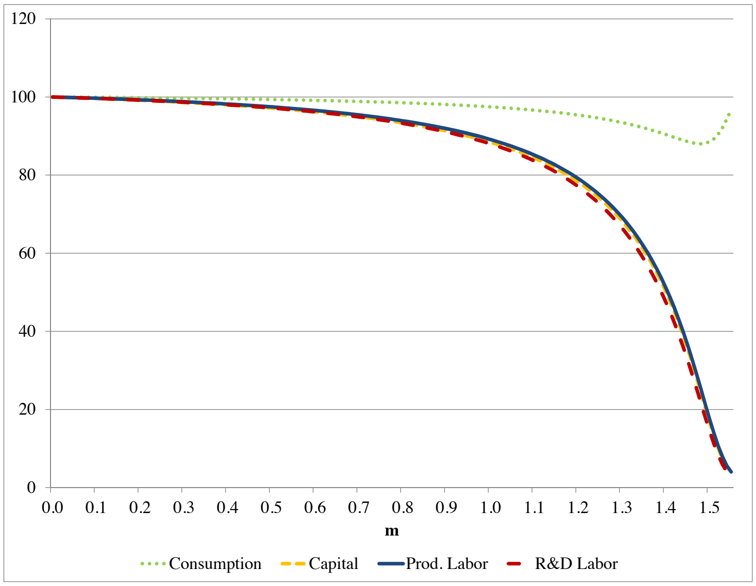
Figure 7 shows how impatient households change their steady-state levels of labor and consumption for different values of m, while Figure 8 shows the effects on patient households.[8]

Figure 7: Effects of Varying m on Impatient Households



[8]For Figure 8, the values for all variables are normalized so that they equal 100 when $m = 0$.

Figure 8: Effects of Varying m on Patient Households



A striking result is how the effect on the growth rate accelerates when households are allowed to be significantly underwater on their debt. To understand this result, consider the model when m is below one. When impatient households are allowed to borrow more, they do so (at the steady state), and their steady-state consumption falls accordingly. They respond to this both by supplying more productive labor and acquiring more capital. With a LTV ratio of less than one, any additional capital increases debt less than one-to-one and increased capital thus has a positive effect on the impatient households' wealth. This secondary effect dampens the overall reduction in wealth and, as a result, all variables exhibit relatively small changes, including the growth rate.

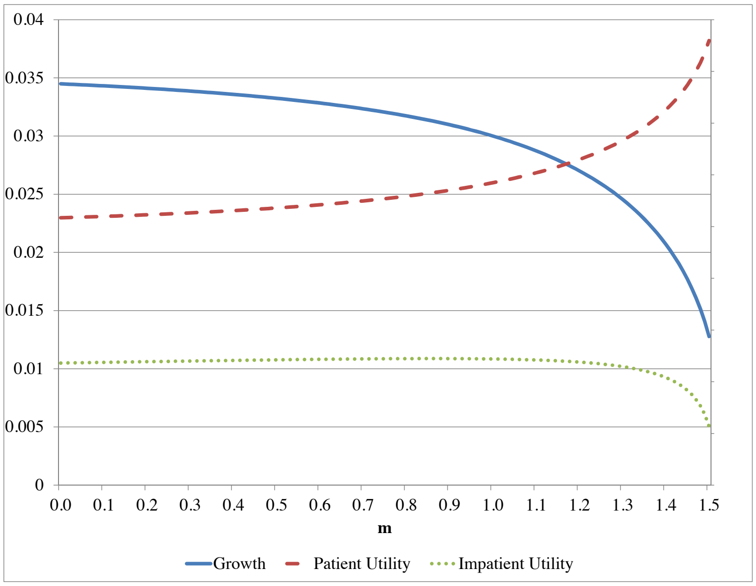
Now suppose that m is above 1. Once again, increased access to credit reduces the impatient households' wealth and increases the impatient households' capital. Now, however, an extra unit of capital results in a more than one-to-one increase in debt. If m is far enough above 1, the effect of extra debt payments will overcome that of the positive marginal product of capital, so that more capital actually reduces the impatient households' wealth. The initial effect on wealth is now amplified instead of dampened so that all variables, including the growth rate, exhibit much larger changes in response to varying m.

Impatient household individual welfare and a Rawlsian social welfare function are maximized when $m = 0.88$. Because growth is now increasing in m, impatient households no longer are better off without any debt even though no debt eliminates the negative wealth effect. Patient household welfare and a Utilitarian social welfare function are highest when $m = 1.55$ (the largest value in this calibration).

3.3 Simulation 3.3: $\epsilon = 1.1$, $\eta = 3$, $\mu = 0.0148$

Our first two simulations assume that labor supply is highly elastic. We now consider a variation of Simulation 3.1 where we continue to assume that $\epsilon = 1.1$, but where labor supply is now inelastic and calibrated at $\eta = 3$. Recall, a low ϵ implies that households do not have a strong preference for smoothing their labor supply across types. Figure 9 reports the results:

Figure 9: Effects of Varying m on Growth and Utility

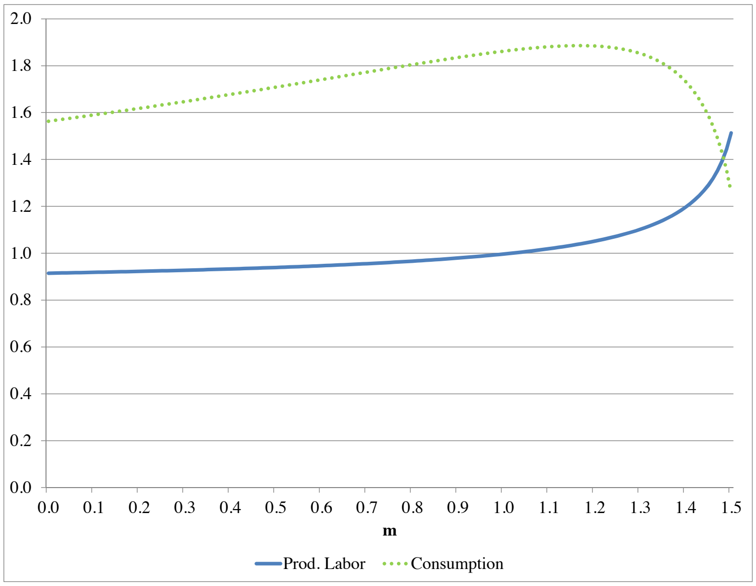


The growth rate is 3.45% at $m = 0$ and is 3.00% at $m = 1$. Growth decreases to around 0.75% as access to credit increases up to the point where $m = 1.50$.

The mechanisms for this simulation are qualitatively the same as for 3.1. Impatient households again supply almost no R&D. As patient households become wealthier, they substitute away from R&D (and productive labor) towards leisure, and growth declines. But because labor supply is less elastic, the changes in R&D and growth are less dramatic, but still large, as compared to 3.1.

Figure 10 shows the effects on impatient households. As in Simulation 3.1, impatient household labor supply to the R&D sector is basically zero.

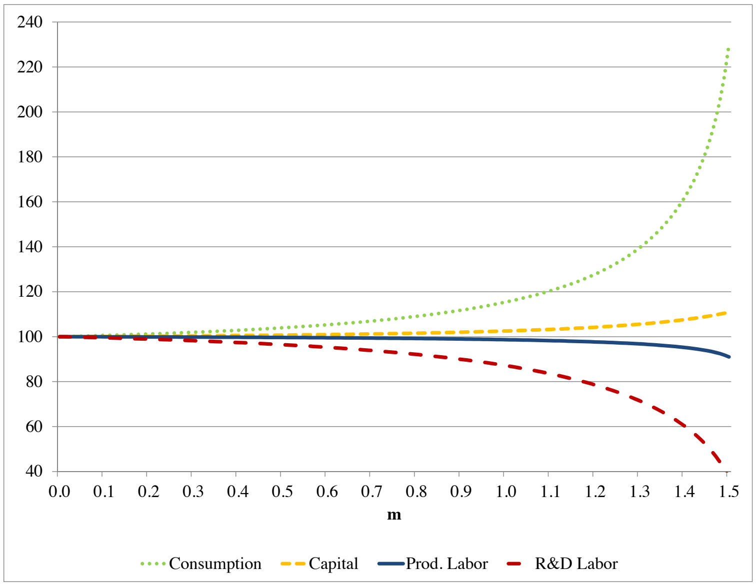
Figure 10: Effects of Varying m on Impatient Households



As m increases, impatient households once again acquire more capital and supply more R&D labor. However, the change in the growth rate is driven by patient households, as shown in Figure 11:[9]

[9] Again, the values of the variables are normalized to equal 100 when $m = 0$.

Figure 11: Effects of Varying m on Patient Households



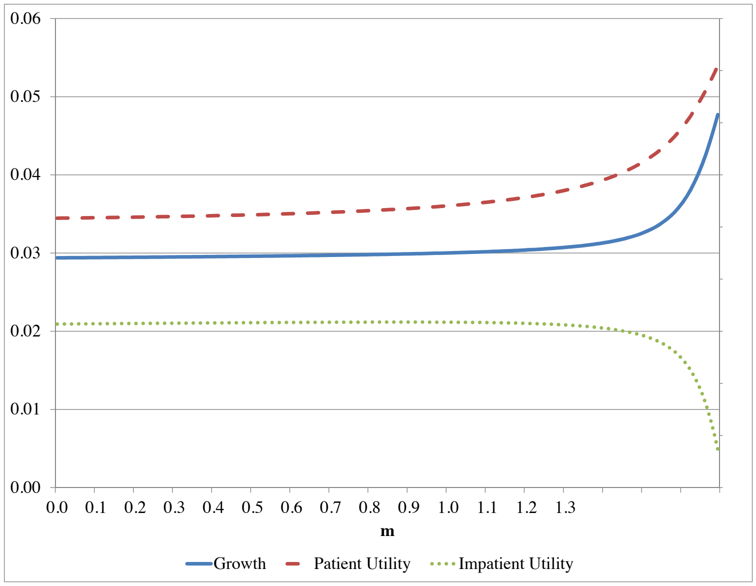
Patient households again supply less R&D and productive labor, but their consumption increases due to the increase in wealth from debt payments. Now, the decline in patient households' R&D causes the decline in growth seen in Figure 9.

Patient household utility is increasing throughout this range of m. Impatient household utility peaks at $m = 0.86$. A Rawlsian social welfare function again tracks the impatient household utility in this simulation. A Utilitarian social welfare function more closely tracks the patient household utility. Once again, the optimal value of m depends on the social welfare function specification but lies between 0.86 and 1.50.

3.4 Simulation 3.4: $\epsilon = 10$, $\eta = 3$, $\mu = 0.0044$

In the last simulation, labor supply is less elastic and households prefer to smooth their labor between production and R&D, as in Simulation 3.2. The results here are similar to that of Simulation 3.2, except that the effects are dampened due to the reduction in labor supply elasticity. Figure 12 shows the impact of a change in m on growth and utility:

Figure 12: Effects of Varying m on Growth and Utility



Growth increases from 2.94% when $m = 0$ to 4.77% when $m = 1.69$. The growth is driven by an increase in impatient household labor supply to the production sector, and as a result of the preference for smoothing labor across types, R&D labor supply also increases. Patient household utility is increasing as access to credit increases. Impatient household utility begins to decline significantly once the LTV ratio exceeds one.

Impatient household utility and Rawlsian social welfare are maximized when $m = 0.87$. The LTV ratio is much higher at $m = 1.69$ (the largest possible value) when maximizing patient household utility. Utilitarian welfare is maximized when $m = 1.61$.

4 Temporary Shocks

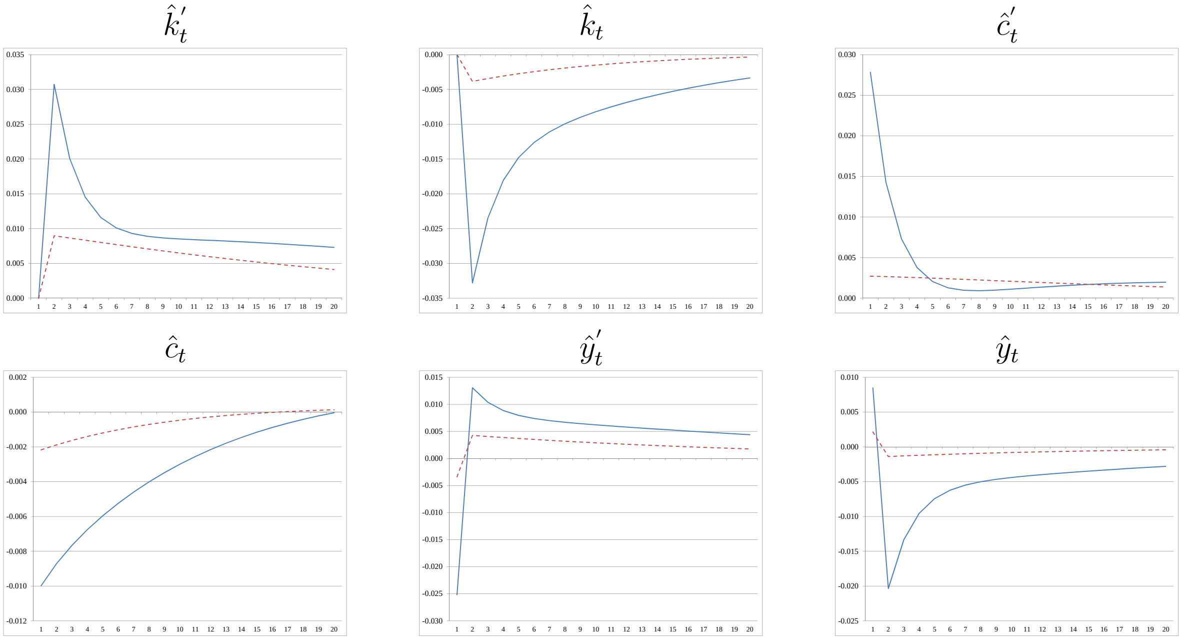
We use the well known method of Blanchard and Kahn (1980) to analyze the dynamics of a linearized version of the model around its steady state. For all of the calibrations reported in Section 3, the steady state is determinate, implying that a single stationary rational expectations equilibrium exists in the neighborhood of the steady state. We now illustrate how the model responds to one-time innovations to credit (m_t), productivity (Z_t), and preferences (U_t).[10]

We have shown that permanent changes to debt have very different effects at the steady state depending on the calibrated value of ϵ. Notably, the model's dynamics around the steady state, including to temporary debt shocks, appear to be largely independent of ϵ. Below, we report impulse response functions using the calibration from Simulation 3.1. The results for Simulation 3.2 are very similar.

We set the AR(1) coefficient equal to 0.95 for the shocks to productivity and debt, while assuming shocks to preferences are iid. We begin by reporting the response to an innovation that increases m by 0.01. We report results for a low debt ($m = 0.1$) case and a high debt ($m = 0.9$) case. The response of each variable is transformed into percentage deviations from the steady state (indicated with a "hat").

[10]In simulating the model, we continue to impose that that the credit constraint is binding. Although this is necessarily true at the steady state, it may not be the case at all times. Iacoviello (2005) examines the effect of making this assumption in a related model and finds that it is small.

Figure 13: Response to Debt Shock
Solid: $m = 0.9$, Dashed: $m = 0.1$

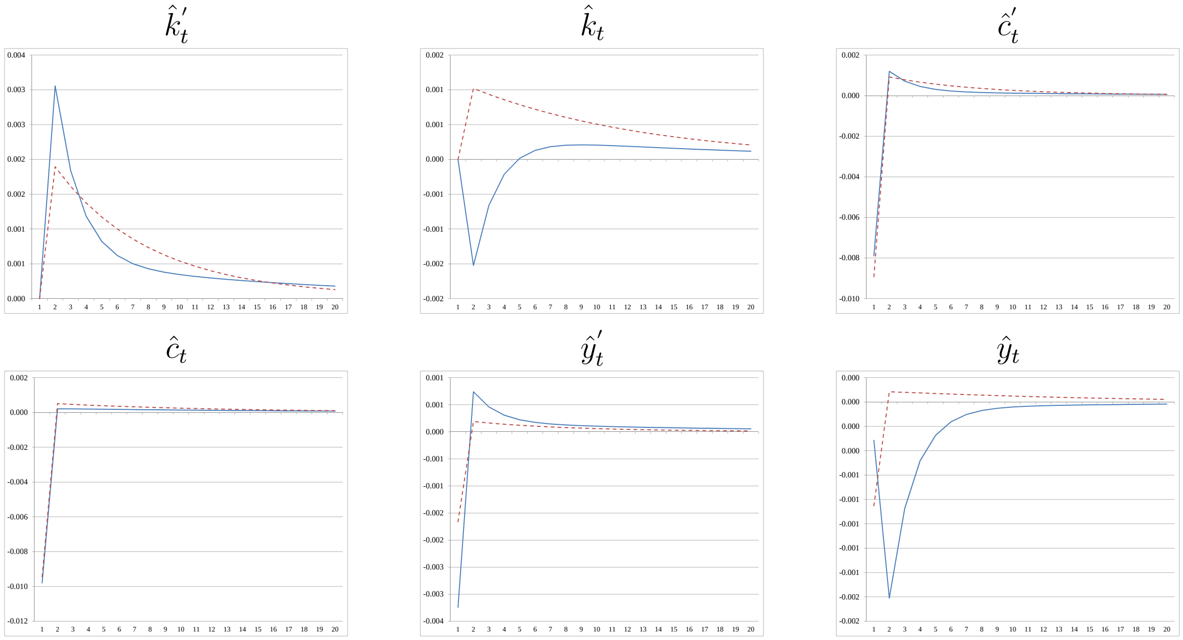


With increased access to credit, impatient households borrow more, resulting in additional consumption. Additional access to credit also incentivizes capital accumulation, which results in increased production from the impatient households. The effects are reversed for patient households. With more of their income being lent to the impatient households, patient households reduce their consumption, capital, and production. Patient households also reduce their supply of R&D, but this results in only a very small decline in the growth rate of trend productivity (≤ 0.00012).

The effects are much larger when the steady state debt level is high. This is consistent with the results of Section 3, which show that the effects on all variables of changing m was larger for $m = 0.9$ as opposed to $m = 0.1$. The initial increase in access to credit results in more capital accumulation which, then allows for even more debt, amplifying the total increase in credit. This effect, and thus the total increase in debt, is larger for the higher steady-state value of m which causes all of the IRFs to have larger magnitudes at $m = 0.9$.

We now consider a one time increase of 0.01 to U_t, suggesting reduced utility from consumption.

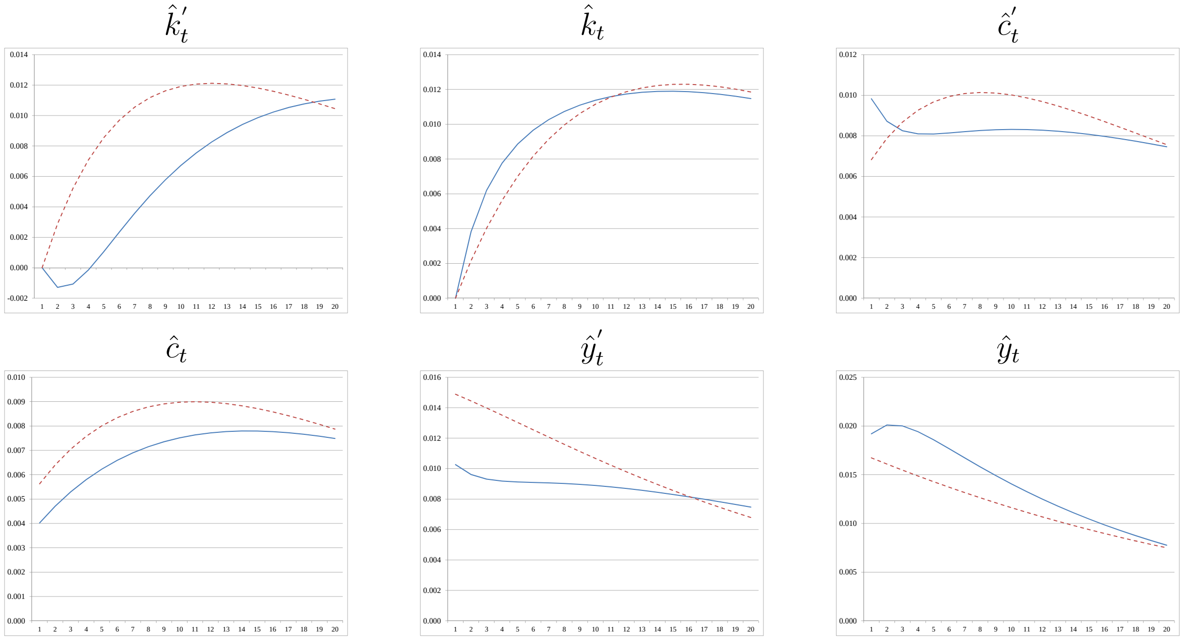
Figure 14: Response to Preference Shock
Solid: $m = 0.9$, Dashed: $m = 0.1$



As expected, both types of households initially reduce their consumption. Impatient households increase their capital stock and initially decrease their output. The effect on patient households depends on m. If m is large, then the shock also causes a large increase in debt. Patient households then substitute away from capital accumulation and their output also falls. If m is small, however, then so is the increase in debt. Faced with lower consumption, patient households now acquire additional capital, which results in more output.

Finally, we consider a transitory innovation to productivity equal to 0.01.

Figure 15: Response to Productivity Shock, Case 1
Solid: $m = 0.9$, Dashed: $m = 0.1$



These effects are unsurprising and do not substantially depend on m. Both types of households increase their capital, consumption, and output. For both shocks to preferences and productivity, g_t exhibits only very small changes.

5 Conclusion

The events of the Great Recession have helped spur increased attention to the macroeconomic effects of limited access to credit. So far, most research has focused on how credit constraints may contribute to short-term volatility. This paper, however, provides conditions where varying households' access to credit affects not just the level of output, but its steady-state growth rate. We have shown that if labor supply is elastic, then these effects may be very large and that the direction of the effect depends on whether households prefer to smooth their labor supply between R&D and productive labor.

Throughout the paper, we have not taken a firm position on the appropriate calibration of labor supply elasticity and the desire of households to smooth their labor supply. The former calibration remains exceptionally controversial. We note that if the reader finds the magnitude of the effects from Simulations 3.1 and 3.2 to be implausibly large, then our results may be seen as an additional piece of support for a more inelastic labor supply which reduces the magnitude of the effects on the growth rate. Because the second calibration is novel to our paper, we are unaware of other work that illuminates its correct value. Empirical work estimating its value might pin down the direction of how access to credit affects growth and is left for future research.

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